

# Wealth Fluctuations and Risk Preferences: Evidence from U.S. Investor Portfolios

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## Abstract

Using data on the portfolio holdings and income of millions of U.S. retirement investors, I find that positive and persistent shocks to income lead to a significant increase in the equity share of investor portfolios, while increases in financial wealth due to realized returns lead to a small decline in the equity share. In a standard homothetic life-cycle model with human capital and constant risk aversion, the portfolio responses to these two wealth shocks should be of equal magnitude and opposite sign. The positive net effect in the data is evidence for risk aversion that decreases in total wealth. I estimate a structural life-cycle consumption and portfolio choice model that accounts for inertia in portfolio rebalancing and matches the reduced-form estimates with a significant degree of non-homotheticity in risk preferences, such that a 10% permanent income growth leads to an average decrease in risk aversion by 1.7%. Decreasing relative risk aversion preferences concentrate equity in the hands of the wealthy and double the share of wealth at the top of the wealth distribution.

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# 1 Introduction

Does relative risk aversion decrease with wealth? The relation between wealth and risk aversion is a key ingredient in any portfolio choice model or macro-finance model that deals with the dynamics of saving and investment over the life cycle or with cross-sectional differences between households. Traditional portfolio choice models and macro-finance models typically assume constant relative risk aversion (CRRA) preferences. An alternative that has gained popularity is decreasing relative risk aversion (DRRA) preferences. In portfolio choice theory, DRRA preferences offer a potential explanation for stylized facts of the data: a relatively flat or even upward-sloping equity share profile over the life cycle and a positive cross-sectional correlation between wealth and equity shares (see, for example, [Carroll, 2002](#); [Wachter and Yogo, 2010](#)). In macro finance, models where risk aversion changes with wealth have had success in matching asset pricing moments ([Constantinides, 1990](#); [Bakshi and Chen, 1996](#); [Campbell and Cochrane, 1999](#); [Wachter, 2006](#); [Verdelhan, 2010](#)) and explaining the joint dynamics of asset prices and business cycle fluctuations ([Jermann, 1998](#); [Boldrin, Christiano, and Fisher, 2001](#)).

Measuring the relationship between wealth and risk aversion in micro data is challenging. First, due to unobserved heterogeneity across investors, this relationship needs to be identified from individual changes over time, which places high demands on the data. Prior findings on the effect of financial wealth on risk taking have been mixed, and depend on the instrument for financial wealth that is used.<sup>1</sup> Second, the effect of financial wealth on risk taking is in itself not informative about DRRA ([Wachter and Yogo, 2010](#)). Since the riskiness of human capital affects optimal portfolio choice, the relative proportion of human capital to financial wealth is a confounding factor. Even with CRRA preferences, changes to financial wealth therefore lead to changes in optimal asset allocations. Finally, to the extent that there is reduced-form evidence that is suggestive of DRRA preferences, it is an open question what the quantitative implications of non-homothetic preferences are in a life-cycle model that matches empirical magnitudes.

In this paper, I use detailed panel data on changes in financial profiles for the same individuals over time to test for non-homothetic risk preferences. I find that positive and persistent shocks to income lead to a significant increase in the equity share of investor portfolios, while increases in financial wealth due to realized returns lead to a small decline in the equity share. The positive net effect of these two wealth shocks is consistent with risk aversion that decreases in total wealth. Second, I use these portfolio responses to estimate the parameters of a structural life-cycle consumption and portfolio choice model that accounts for inertia in portfolio rebalancing. The model matches the reduced-form estimates with a significant degree of non-homotheticity in risk preferences, such that a 10% permanent income growth leads to a decrease in risk aversion by 1.7%. Third, I find that decreasing risk aversion has important quantitative implications for

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<sup>1</sup>[Brunnermeier and Nagel \(2008\)](#) and [Chiappori and Paiella \(2011\)](#) do not reject the absence of an effect of financial wealth on risk taking. On the other hand, [Calvet, Campbell, and Sodini \(2009\)](#), [Calvet and Sodini \(2014\)](#), and [Paravisini, Rappoport, and Ravina \(2017\)](#) find evidence in support of a positive relation between financial wealth and risk tolerance. I review these findings below.

wealth inequality. Since wealthier households select riskier positions, they have higher average returns. Due to this heterogeneity in expected returns to financial wealth, the wealth share of the top 1% in the model nearly doubles. The model also implies that rising income inequality in the U.S. over the past three decades has led to an increase in demand for equity that is equivalent to a 15% decline in the equity premium.

How can the relation between wealth and risk aversion be estimated from micro data? Households face two main types of wealth shocks: income growth and returns on financial assets. By observing the portfolio responses to these two wealth shocks, the effects of changes in the composition in wealth can be separated from the effects of changes in the overall level of wealth. In a homothetic life-cycle model where risk aversion is constant, only the composition of wealth matters for optimal allocations and not the level of wealth. This means that the relevant state variable is the ratio of human capital to financial wealth. In that case, the portfolio responses to income growth and portfolio returns exactly offset each other – up to log-linear approximation – and therefore add up to zero. A positive net effect is evidence for DRRA preferences. Thus, DRRA can be detected by measuring the joint effects of income growth and realized portfolio returns on portfolio allocations.

To measure these portfolio responses, I use a dataset that contains individual portfolio holdings, trades, income, and demographic characteristics of millions of U.S. retail investors with trillions of dollars in investable wealth. The sample covers annual observations between 2006 and 2018 and therefore spans various market conditions. I restrict attention to a subsample that is representative of the data and a subset of the overall U.S. population: Retirement Investors (RIs), which are “typical” American investors that have retirement assets in the middle 80% of the age-adjusted redistribution of retirement wealth, and for whom retirement savings are the main form of investable wealth.

In response to an increase in income growth, retirement investors increase their allocation towards equity, and reduce allocations to bonds and cash-like securities. This relation is driven by investor-driven portfolio changes through trading, not by ex-ante differences in portfolios and market fluctuations, and is robust to including various sets of demographic controls and employer-year fixed effects. While the variation in the data is predominantly coming from income changes within jobs, I find similar effects for the subset of investors that had a job change. The effects of income growth on equity share changes are long-lasting – portfolio allocations do not revert back to the initial composition. As a result, there is a cross-sectional correlation between income levels and equity shares.

Theory predicts that investors should respond differently to persistent income shocks than to transitory income shocks. Indeed, the positive relation between income growth and equity share changes is stronger over longer horizons, where persistent shocks are a relatively bigger component of total income growth. I measure the effect of persistent income growth on portfolios by instrumenting income growth by the long difference of lead income and lag income. The effect of persistent income growth on portfolio equity shares is more than double the effect of overall

income growth. The coefficient does not change when controlling for employer-year fixed effects or annual effects by observable characteristics. Although the relation between income growth and changes in portfolio equity shares is highly statistically significant, the magnitude of the effect is modest, even when focusing on persistent income growth. The baseline estimate implies that a ten percent increase in the persistent component of labor income leads to an increase in equity share by 0.4 percent.

I find considerable heterogeneity in the effect of income growth on equity shares due to infrequent portfolio adjustment. It is well known that many investors irregularly update their portfolios, in particular in retirement accounts. A large share of the sample stick with the default allocation and only a small percentage of the sample rebalance their portfolio in any given year. First, I find that investors without initial portfolios at the default allocation have a much stronger response to income shocks. Second, the effects are driven by a small share of the sample that reallocate their portfolios. Conditional on having a significant portfolio turnover, a ten percent increase in persistent income leads to an increase in equity share by 3.0 percent. Third, the effect size is strongly increasing in the number of web visits that investors make during the year, suggesting that inattention might be an important channel through which infrequent adjustment arises.

To quantify the effect of income on targeted equity holdings and to separate the effects of asset fluctuations from investor reallocation decisions, I estimate a partial adjustment model similar to [Calvet et al. \(2009\)](#). The model quantifies the speed of adjustment in portfolios and changes in desired portfolio allocations. In line with baseline statistics on trading behavior, I find that the asset return-driven component captures over 80 percent of overall portfolio changes. Controlling for the effects of infrequent adjustment and for aggregate market movements, the effect of idiosyncratic portfolio returns on risk taking is slightly negative. The positive net effect of income growth and portfolio returns on equity shares conflicts with the prediction of a standard homothetic life-cycle model and is consistent with risk aversion that decreases in total wealth.

In a second step, I investigate the quantitative implications of these findings by specifying and estimating a realistic life-cycle consumption and portfolio choice model. In the model, agents receive labor income subject to uninsurable idiosyncratic shocks during their working life and allocate their savings towards a risky and a riskless asset. The model has three important ingredients: a preference specification that generalizes CRRA preferences and can account for non-homothetic risk preferences, human capital with countercyclical tail risk in permanent income, and infrequent portfolio rebalancing. Since in the data I find that trading activity is largely determined by outside factors – the predictive power of changes in financial circumstances on investor reallocation activity is low and the hazard function of portfolio adjustment is flat, I model rebalancing to be time dependent rather than state dependent.<sup>2</sup> The parameters of the model that are estimated through indirect inference are the rate of time preference, baseline risk aversion, non-homotheticity in risk aversion, and the frequency of portfolio rebalancing. To match

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<sup>2</sup>[Giglio, Maggiori, Stroebel, and Utkus \(2019\)](#) arrive at a similar conclusion for the effect of beliefs.

the additional empirical fact that permanent-income rich households save disproportionately more than poor households and therefore the cross-sectional relation between consumption and permanent income is concave (Straub, 2019), I allow for heterogeneity in discount factors that is correlated with permanent income levels. The model is able to fit the regression evidence on portfolio changes in response to wealth shocks, jointly with the age profile of savings to income and the average equity share at age 50 from SCF. Key to this result are a significant elasticity of risk aversion with respect to wealth and a model of human capital that accounts for cyclicity in high order moments of income growth.

Third, I find that the model has important long-run implications for inequality and asset prices. Since risk aversion decreases in wealth, richer households choose to invest a larger share of their portfolios in equity than poorer households. Wealthier households therefore have higher returns on average. This positive correlation between wealth and average portfolio returns matches empirical patterns (Fagereng, Guiso, Malacrino, and Pistaferri, forthcoming).<sup>3</sup> The estimated non-homotheticity in risk preferences thus implies a two-way relation between wealth and equity demand that has important implications for inequality. The range of expected returns by net worth, conditional on age, nearly spans the full equity premium: households in the lowest percentile of the net worth distribution invest the majority of their financial wealth in the risk-free asset, while households in the top of the net worth distribution invest only in equity. By targeting the within-person portfolio responses to wealth shocks and the cross-sectional relation between consumption and permanent income, the model generates an (untargeted) wealth distribution with large inequality: the wealth share of the top 1% is 37.0%. An important contributor to this large wealth inequality is that equity holdings are concentrated in the hands of the rich. In an alternative estimation of the model, I show that if risk preferences were CRRA, the top 1% wealth share would only be 20.4%.<sup>4</sup>

Finally, I use the model to examine the effects of rising income inequality in the United States over the past few decades on wealth inequality and asset demand. An important force behind increased income inequality is an increase in the dispersion of permanent income levels of new cohorts (Güvenen, Kaplan, Song, and Weidner, 2018). I calibrate the dispersion in initial income to match the Gini coefficient of income in the SCF and compare simulations of the model with levels of income inequality in 1989 and 2016. The top 1% wealth share rises from 22.0% to 37.0%. I calculate the change in the equity premium that offsets the change in demand in the model such that the aggregate equity share reverts back to its original value.<sup>5</sup> The model suggests that inequality over the past few decades has led to an increased demand for equity that is equivalent to a decrease in the equity premium of 0.69 percentage points (15% of the equity premium in the model).<sup>6</sup>

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<sup>3</sup>Since all agents in the model have access to two assets, a risk-free asset and a stock market index, these patterns are fully driven by differences in risk-taking behavior as opposed to differences in technologies. Fagereng et al. (forthcoming) find that average returns to net worth are different even within narrow asset classes.

<sup>4</sup>Similarly, in a version of the model where the equity premium is set to zero, the top 1% wealth share is 19.4%.

<sup>5</sup>Catherine (2019) runs a similar exercise to calculate the effects of cyclical skewness in income growth on risk premia.

<sup>6</sup>Evidence on changes in the equity risk premium over this period is mixed. Pástor and Stambaugh (2001), Fama

**Literature.** Portfolio choice theory prescribes how investors should allocate their financial wealth across asset classes under a wide set of assumptions on preferences, human capital endowments, and other constraints or financial characteristics. In the absence of non-tradable labor income, the optimal equity share of a CRRA investor is constant (Samuelson, 1969; Merton, 1969). With human capital, what matters is the relative proportion of human capital to financial wealth; the overall level of wealth plays no role. Risk-free human capital implies that the demand for equity in investment portfolios should increase in the share of human capital in total wealth. As human capital diminishes with age and as financial wealth grows, the share of financial wealth invested in equity decreases over the life cycle (Jagannathan and Kocherlakota, 1996). This insight extends to many quantitative life-cycle models where human capital has limited stock-like properties.<sup>7</sup> Empirical findings on investor portfolios pose several challenges to these theoretical predictions. First, the portfolio equity share of investors that participate in financial markets tends to be relatively flat or even upward sloping over the life cycle (Guiso, Haliassos, and Jappelli, 2002; Ameriks and Zeldes, 2004). Second, the average portfolio equity share of participants is relatively low compared to optimal allocations in a model with bond-like human capital and moderate risk aversion. Third, it has been well documented that there is a positive relation between wealth and risk taking in the cross section of households (see e.g. Heaton and Lucas, 2000; Carroll, 2002). Two important variations on the traditional setup of life-cycle models that address these facts and have been studied extensively are deviations from CRRA preferences and alternative properties of human capital.

First, Wachter and Yogo (2010) show the appeal of DRRA preferences for matching equity share profiles over the life cycle and the cross-sectional relation between wealth and asset prices. However, empirical evidence on the link between wealth and risk preferences is mixed. Since cross-sectional differences in portfolios could be due to differences in risk tolerance, influenced for instance by socioeconomic status, this link cannot be estimated in a simple cross-sectional OLS regression. Several papers have looked at the effects of financial wealth on risk-taking behavior in panel data. Chiappori and Paiella (2011) use Italian data to regress changes in risky shares on changes in wealth and find support for a CRRA specification. To address measurement error in the joint measurement of financial wealth and portfolio allocations, Brunnermeier and Nagel (2008) instrument wealth growth by income growth and find a negative relation between wealth changes and risky portfolio shares, while Calvet et al. (2009) instrument wealth changes with return realizations and find a positive effect of wealth growth on portfolio risk taking. Calvet and Sodini (2014) use a different identification strategy by running a cross-sectional regression of risk taking on financial wealth in a dataset of Swedish twin investors and controlling for twin fixed

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and French (2002), and Jagannathan, McGrattan, and Scherbina (2000) found that the equity premium decreased in the decades leading up to the 2000s. Duarte and Rosa (2015) and Caballero, Farhi, and Gourinchas (2017) argue that the equity premium has increased over the most recent decades.

<sup>7</sup>See e.g. Bertaut and Haliassos (1997), Heaton and Lucas (1997), Gakidis (1998), Viceira (2001), Campbell, Cocco, Gomes, and Maenhout (2001), Cocco, Gomes, and Maenhout (2005), Gomes and Michaelides (2005), Gomes, Kotlikoff, and Viceira (2008), and many others.

effects.<sup>8</sup> They find that portfolio shares in risky assets are strongly increasing in financial wealth. Lastly, [Paravisini et al. \(2017\)](#) find that risk aversion increases in response to a negative housing wealth shock. By looking at the reduced-form effect of financial wealth on risk-taking behavior, these papers do not explicitly make the distinction between (1) the effect of overall wealth on risk-taking behavior, and (2) the effect of a changing composition of total wealth. Only [Calvet and Sodini \(2014\)](#) have important evidence on portfolio risk taking by both financial wealth and human capital, identified from twin regressions. They find an effect of human capital on risky asset allocations that is positive and significant for identical twins. However, the measured effect of human capital is much smaller in magnitude than the effect of financial wealth. These findings are hard to reconcile with a portfolio choice model that matches the empirical properties of labor income risk. In contrast, I find a strongly significant positive effect of labor income on equity shares and a smaller negative effect of portfolio returns.

Second, several papers have argued that human capital has more stock-like properties than what is implied by the low correlation between income growth and stock returns. [Benzoni, Collin-Dufresne, and Goldstein \(2007\)](#) document cointegration between wages and dividends and show that this cointegration alters the life-cycle profile of equity shares. Other papers have explored variation in idiosyncratic income risk. [Storesletten, Telmer, and Yaron \(2007\)](#) and [Lynch and Tan \(2011\)](#) consider countercyclical variation in volatility, which is absent in U.S. administrative data ([Güvenen, Ozkan, and Song, 2014](#)). [Catherine \(2019\)](#) explores the role of cyclical skewness in labor income and finds that a model with large countercyclical tail risk can generate an upward-sloping age profile of equity shares.<sup>9</sup> I show that accounting for tail risk in labor income is important for matching micro evidence on individual portfolio changes. However, only in combination with a significant degree of non-homotheticity in risk preferences can the model match the empirical findings.

Finally, since panel data on portfolios is scarce, the portfolio choice literature has primarily relied on cross-sectional (survey) evidence on investor portfolios to calibrate or estimate life-cycle model parameters from differences across investors with different ages and other characteristics. Recent papers by [Fagereng, Gottlieb, and Guiso \(2017\)](#) and [Calvet, Campbell, Gomes, and Sodini \(2019\)](#) improve on identification by using detailed administrative panel data to estimate structural preference parameters of life-cycle models. These papers assume constant risk aversion and target age profiles of wealth accumulation and equity shares. I contribute to this literature by allowing for wealth-dependent risk aversion and targeting individual portfolio changes in response to wealth shocks.

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<sup>8</sup>A caveat is that even with perfect controls for background heterogeneity, cross-sectional regressions in levels potentially suffer from reverse causality.

<sup>9</sup>Countercyclical tail risk in non-diversifiable income risk also has important asset pricing implications. These implications are studied by [Constantinides and Ghosh \(2017\)](#) and [Schmidt \(2016\)](#).



## 2 Motivation of Empirical Analysis

This section motivates the empirical analysis in Sections 4 and 5 by illustrating the implications of portfolio choice theory for changes in risky asset holdings in response to wealth changes. The objective is to show how the combined effects of income growth and asset returns on portfolios are informative about risk preferences while accounting for human capital endowments. I review a stylized two-period model and discuss how the insights of the stylized model extend to a more general life-cycle portfolio choice model.

### 2.1 Stylized Model Setup

I analyze a simple two-period model that builds on [Campbell and Viceira \(2002\)](#). A household is endowed with financial wealth  $W$  at time  $t = 0$  that can be invested in a risky and a risk-free asset. Consumption takes place at time  $t = 1$ , when all wealth  $W_1$  is consumed. The household chooses its portfolio to maximize expected utility at  $t = 1$ :

$$\max \frac{\mathbb{E}_0 [(W_1 - X_1)^{1-\gamma}]}{1-\gamma}, \quad (1)$$

where  $\gamma$  is the curvature of the utility function and  $X_1$  is a minimum subsistence or habit level. When  $X_1 = 0$ , preferences are CRRA. The case of  $X_1 > 0$  implies that risk aversion decreases in wealth.

Denote the time-1 stochastic gross return on the risky asset by  $R^e$ . The risk-free asset pays a fixed return  $R^f$ . At time  $t = 1$ , households also receive labor income  $P_1$ .<sup>10</sup> Let  $\phi$  be the correlation between labor income and the return on the risky asset. In addition, assume for simplicity here that human capital only loads on aggregate risk and is not subject to idiosyncratic risk.<sup>11</sup>

$$P_1 = P(\phi R^e + (1 - \phi)R^f), \quad (2)$$

where  $P$  is the present value of labor income.

### 2.2 Portfolio Choice

Investors decide at time  $t = 0$  on the share of financial wealth  $W$  that is invested in the risky asset,  $\theta$ . Terminal wealth is given by

$$W_1 = W(R^f + \theta(R^e - R^f)) + P_1. \quad (3)$$

Since labor income is tradable and portfolios are unconstrained, the problem can be reduced to an investment decision on the allocation of total wealth  $W + P$  – the optimal allocation of financial

<sup>10</sup>For consistency with the rest of the paper,  $P_1$  is interpreted as permanent income.

<sup>11</sup>The presence of idiosyncratic income risk does not affect the results here since the focus is on deriving a simple log-linearized solution. Idiosyncratic risk will play a key role in the full quantitative life cycle model in Section 6.



wealth then follows trivially from the desired allocation of total wealth by adjusting for the relative proportion of human capital to financial wealth.

I derive an approximate expression for the optimal portfolio allocation  $\theta$  using a log-linear approximation to the Euler equation. Let  $\mu_e = \mathbb{E}[\log R^e]$ ,  $r_f = \log R^f$ , and  $\sigma_e^2 = \text{Var}[\log R^e]$ .

**Proposition 1.** *The optimal portfolio share as a fraction of financial wealth is, up to a log-linear approximation, given by*

$$\theta = \bar{\alpha} + (\bar{\alpha} - \phi) \frac{P}{W} - \bar{\alpha} \left( 1 + \frac{P}{W} \right) \frac{X}{W + P}, \quad (4)$$

where  $\bar{\alpha} = \frac{\mu_e - r_f + \frac{1}{2}\sigma_e^2}{\gamma\sigma_e^2}$ .

The proof is in Appendix A.1. A benchmark case is the model without human capital ( $P = 0$ ) and with constant relative risk aversion preferences ( $X = 0$ ), which yields the seminal result from Samuelson (1969) and Merton (1969) that the portfolio equity share satisfies the “myopic” allocation  $\bar{\alpha}$ . With risk-free labor income ( $\phi = 0$ ), the optimal portfolio share is increasing in the ratio of permanent income to financial wealth. More generally, the portfolio is tilted towards risky assets when human capital is safer than the optimal allocation of total wealth ( $\bar{\alpha} > \phi$ ), and tilted towards risk-free assets when human capital is riskier ( $\bar{\alpha} < \phi$ ). A positive subsistence level  $X$  increases effective risk aversion and therefore lowers the optimal allocation towards equity, since a larger share of the portfolio is devoted towards insurance against the fixed subsistence level  $X$ .

### 2.3 Wealth Shocks and Portfolio Allocations

For a fixed subsistence level  $X$ , the optimal portfolio at  $t = 0$  is function of the state variables  $W$  and  $P$ . To guide the empirical analysis on dynamic portfolio choice, I derive comparative statistics of the portfolio share at  $t = 0$ . Suppose that at time  $t = 0$ , cash on hand is given by

$$W = \bar{W}R^{pf} + P, \quad (5)$$

where  $R^{pf}$  is the return on a pre-determined portfolio that is realized at  $t = 0$ .

Let  $X = \bar{X}R^f$  and  $P = \bar{P}R^p$ , where  $\bar{X}$  and  $\bar{P}$  are present values of claims to  $X$  and  $P$  prior to  $t = 0$ , respectively, and  $R^p$  is the realization of permanent income growth at  $t = 0$ . Another log-linearization around  $R^{pf} = R^p = R^f$  generates an expression for the effects of income and realized portfolio returns on portfolio allocations.

**Proposition 2.** *The optimal portfolio is, up to a log-linear approximation, given by*

$$\log \theta = \text{const} + \underbrace{(\kappa_1\lambda_1 + (1 - \kappa_2)\lambda_2)}_{b_1} \log P + \underbrace{(-\kappa_1\lambda_1 + \kappa_2\lambda_2)}_{b_2} \log R^{pf}, \quad (6)$$

where  $\kappa_1, \kappa_2 \in (0, 1)$ , and

$$\lambda_1 = c_1(\bar{\alpha} - \phi), \quad c_1 > 0 \quad (7)$$

$$\lambda_2 = c_2\bar{X}, \quad c_2 > 0. \quad (8)$$

It follows that

$$\begin{cases} b_1 + b_2 = 0 & \text{if } \bar{X} = 0 \\ b_1 + b_2 > 0 & \text{if } \bar{X} > 0. \end{cases} \quad (9)$$

The coefficients  $b_1$  and  $b_2$  capture the effects of income growth and portfolio returns on equity allocations, respectively. Both types of wealth shocks affect optimal allocations through two channels: a human capital channel and a risk aversion channel. The effect of human capital is determined by  $\lambda_1$  and is driven by the equity exposure of human capital,  $\phi$ . With relatively safe (risky) human capital,  $\lambda_1 > 0$  ( $\lambda_1 < 0$ ). The effect of DRRA preferences is determined by  $\lambda_2$  and is driven by the degree of non-homotheticity  $\bar{X}$ . With CRRA preferences,  $\bar{X} = 0$  and hence  $\lambda_2 = 0$ . Figure 1a plots the coefficients  $b_1$  and  $b_2$  by  $\phi$ , for  $X = 0$ . The coefficient  $b_1$  is decreasing in  $\phi$ , while  $b_2$  increases in  $\phi$ .

Equation (9) provides a restriction of homothetic preferences on the joint portfolio effects of income growth and portfolio returns. Under homothetic preferences, only the human capital channel is active. This channel generates offsetting implications of human capital and financial wealth on the equity share – up to log-linear approximation – and therefore the two coefficients add up to zero. With DRRA preferences, the net effect is positive.

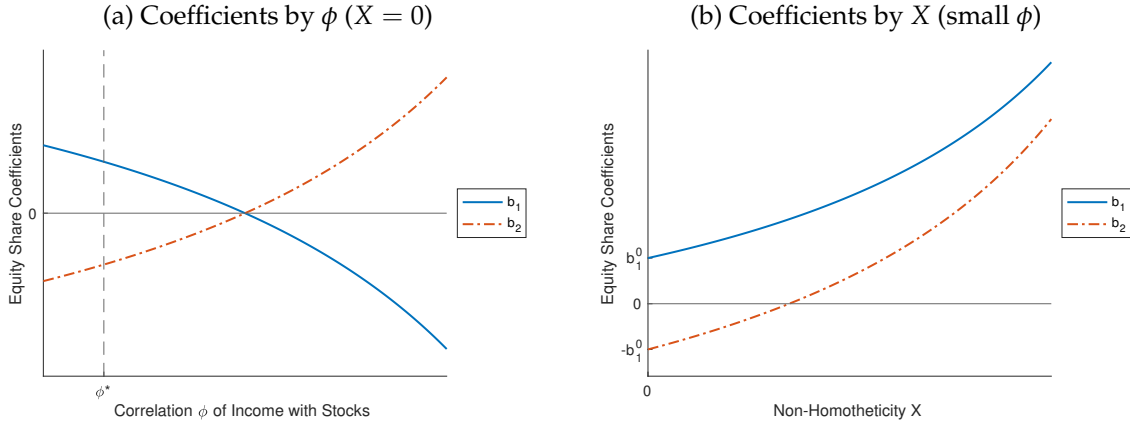
Importantly, the individual effects of fluctuations in either component of total wealth are not informative about risk preferences. Panel 1b plots the coefficients  $b_1$  and  $b_2$  by  $X$ , for human capital that is mostly bond-like (low  $\phi$ ) as in typical empirical estimates. Note that  $b_2$ , the effect of changes in financial assets, can be negative even when risk aversion decreases in wealth (Wachter and Yogo, 2010). Similarly, Figure 1a illustrates that a positive relation between asset returns and equity shares can be generated in a model with CRRA when human capital is sufficiently risky. In order to test for DRRA and identify the key parameters of a life-cycle portfolio choice model, it is therefore crucial to measure the effects of both income growth and financial returns in the data.

## 2.4 Extension to Dynamic Model

The basic implications from the stylized two-period model carry over to a general life-cycle framework. I now introduce three assumptions that are met by many standard life-cycle models and generate a homothetic value function. In that case, the effects of income shocks and asset returns on optimal portfolio allocations offset each other (to an approximation).

**Assumption 1** (Homothetic preferences). *Agents have Epstein-Zin preferences with a constant elasticity of intertemporal substitution  $\psi$  and coefficient of relative risk aversion  $\gamma$ . The value function at death, capturing bequest motives, is proportional to cash on hand  $W_{it}$ .*

Figure 1: Coefficients in Stylized Model



Notes: This figure illustrates the coefficients  $b_1$  and  $b_2$  on income and portfolio returns, respectively, in the stylized model, as a function of the riskiness of human capital  $\phi$  and non-homotheticity in risk preferences  $X$ .

**Assumption 2** (Income with permanent and transitory shocks, linear taxes). *Pre-tax income  $Y_{it}$  can be written as product of a deterministic component in age  $G_t$ , a permanent component  $P_{it}$ , and an idiosyncratic component  $e^{\epsilon_{it}}$ . In retirement, agents receive an income that is a constant fraction of permanent income in the final pre-retirement period. Income taxes are linear:  $Y_{it}^{post} = (1 - \tau)Y_{it}$ .*

**Assumption 3** (Constant investment opportunities). *Investors allocate financial wealth to financial assets that have i.i.d. returns, so that investment opportunities are constant over time. There is either no borrowing constraint, or a borrowing constraint of zero.*

Under these three assumptions, the problem admits a homothetic solution where the level of total wealth is irrelevant for saving rates and portfolio choice, and only the proportions of financial wealth and human capital matter. Thus, the state variables in the homothetic model are age  $t$  and cash on hand relative to permanent income,  $w_{it} \equiv \frac{W_{it}}{G_t P_{it}}$ . The law of motion for  $w_{it}$  is given by

$$w_{i,t+1} = (w_{it} - c_{it})R_{i,t+1}^{pf} \frac{G_t P_{it}}{G_{t+1} P_{i,t+1}} + (1 - \tau)e^{\epsilon_{i,t+1}}, \quad (10)$$

where  $c_{it} \equiv \frac{C_{it}}{G_t P_{it}}$  is normalized consumption.

Let the optimal portfolio be given by  $\theta_{it} = \Theta(w_{it}, t)$ . I consider a log-linear approximation to again find the coefficients  $b_1$  and  $b_2$  on income growth and portfolio returns.<sup>12</sup>

**Proposition 3.** *Under assumptions 1–3, the optimal portfolio in the homothetic dynamic model is, up to a*

<sup>12</sup>Note that in a dynamic model, these coefficients depend on state variables at time  $t$ . I suppress the dependence of the log-linearization constants on time- $t$  information here. In empirical work and for identifying the parameters of the structural model, I will mainly focus on the average values of these coefficients.

log-linear approximation, given by

$$\log \theta_{i,t+1} = k + b_1 \Delta \log P_{i,t+1} + \underbrace{(-b_1)}_{b_2} \log R_{i,t+1}^{pf} + b_3 \epsilon_{i,t+1}. \quad (11)$$

The approximate restriction  $b_1 + b_2 = 0$  under homothetic preferences thus extends to a dynamic life-cycle model. As in the stylized model,  $b_1$  is positive and  $b_2$  is negative in the traditional calibration of the model where income is mostly bond-like (see e.g. Wachter and Yogo, 2010). My empirical strategy will focus on measuring these coefficients  $b_1$  and  $b_2$  in the data. I will then estimate a structural model that targets these empirical moments and quantifies the degree of non-homotheticity in risk preferences.

## 3 Data

### 3.1 Description

I analyze the portfolio behavior of a large sample of individual investors in the United States. For this analysis, I use a dataset of account-level administrative data on financial holdings, transactions, and investor characteristics from a large U.S. financial institution. The data cover anonymized information on all taxable and non-taxable accounts of individual investors at the firm. For these accounts, all balances and security-level portfolio holdings are available at an end-of-year frequency between 2006 and 2018, as well as all inflows, outflows, and transactions at a daily frequency. The data span millions of investors with trillions in financial wealth.

In addition to detailed information on financial portfolio compositions and trades, the data contain information on investor demographics and employment-related variables. The main demographic variables that are covered in the data are age, gender, marital status, and zip code. Employment-related information is available for a subsample of the client base that have an active employment relationship. For this subset, I observe an anonymized employer indicator, employer industry (3-digit NAICS code), employment tenure, and, importantly, gross annual wage income. I annualize all income observations by scaling up part-year incomes to a full-year equivalent. I drop annualized incomes that are below the annual minimum wage at 20 hours per week (\$7540 from 2010). All variables are constructed at the individual level.<sup>13</sup>

To characterize portfolio allocations, all fund and individual security holdings are divided into four main asset categories: equity, fixed income, cash and cash-like assets, and alternative assets. Multi-asset class funds (e.g. target date funds) are split between equity and fixed income in proportion to the observed equity share of the fund. I also compute the market betas of all assets

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<sup>13</sup>While the data include household identifiers that allow spouses to be linked, in most cases only one spouse is observed. Requiring income to be observable, as I do in this paper, further tilts the sample towards households with full information for only one person. For transparency, I therefore run the analysis at the individual level. Where there are multiple individuals that co-own an account, the account is assigned to a single individual by selecting the (oldest) owner with the highest total assets. This yields a unique mapping from accounts to individuals. The results do not change when running the analysis at the household level.

by regressing fund and security excess returns on the market excess return, requiring at least 24 months of return observations. These asset characteristics are complemented by other security-level information, such as international exposures. Appendix A.2 contains additional details on the data construction.

### 3.2 Sample Selection

While the dataset used for this analysis provides a rare opportunity to study detailed portfolio allocations of a large sample of U.S. retail investors, there are two potential limitations of this data. First, even though the data cover a significant share of American retail investors, the sample is obviously selected by holding an investment account at this firm. In particular, most investor wealth is in retirement accounts and few investors have very high net worth. Second, investors may have other investable wealth in accounts at another firm. To address these issues, I follow an approach similar to Meeuwis, Parker, Schoar, and Simester (2019) by restricting the analysis to a subsample of investors that is reasonably representative of “typical” American investors with some retirement savings. To that end, I impose sampling restrictions on age and retirement wealth.

First, I select a sample of investors that are between 30 and 58 years of age. I exclude younger and older investors for two reasons: (i) the youngest age group typically has low levels of investable wealth, while retired investors are underrepresented due to attrition from the sample. I select age 58 as the upper bound because penalty-free withdrawals from retirement accounts can be taken from age 59.5. And (ii), standard life-cycle models are best equipped to capture the wealth accumulation and investment behavior of households at middle ages and tend to perform worse at young age and at retirement.

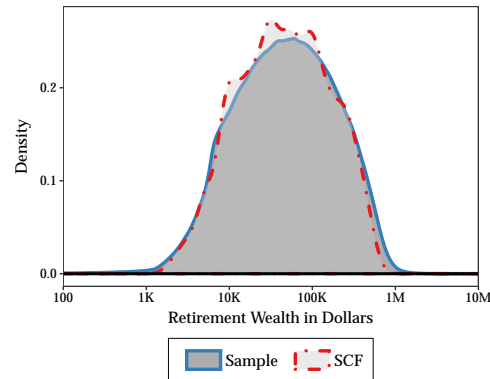
The second restriction I impose is on retirement wealth. In particular, I focus on investors with moderate levels of retirement wealth, labeled as retirement investors (RIs). Specifically, RIs are investors without extremely high or low retirement wealth, defined as all wealth in retirement saving accounts of all types (excluding defined benefit plans and Social Security). The cutoffs are determined using the 2016 SCF. For the sample of working age investors with positive retirement wealth, I run quantile regressions of the log of retirement wealth on a third-order polynomial in age. The 10th and 90th percentiles by age form the cutoffs for selection into the sample. Within the set of investors of age 30 to 58, the sample of RIs captures 40% of the population, 51% of retirement wealth, and 44% of household investable wealth, where investable wealth is defined as the total of money market funds, non-money market funds, individual stocks and bonds, certificate of deposits, quasi-liquid retirement wealth, and other managed accounts.

Finally, estimating the effect of income on portfolio allocations is a key part of the empirical analysis in this paper. I therefore require individuals in the sample to have an active employment relation where income is observable.<sup>14</sup> In particular, I select RIs into the sample in year  $t$  when income is observed at  $\{t - 1, t, t + 1\}$ . There are two main reasons for requiring the lead and lag of income to be observable. First, this restriction limits the impact of measurement error in full-year

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<sup>14</sup>41% of the full RI sample has an income observation. This subsample still spans millions of investors.

Figure 2: Individual Retirement Wealth Distribution



*Notes:* This figure plots the distribution of individual retirement wealth in the sample of RI investors versus the distribution of individual retirement wealth for RI investors in the SCF.

incomes inferred from part-year income observations. Second, I use lead and lag observations of income as instruments when estimating the effects of persistent income growth. As a consequence of this restriction, the final dataset used for the empirical analysis runs from  $t = 2007$  to  $t = 2017$ .

Figure 2 shows that the distribution of retirement wealth in the sample of RIs lines up well with the distribution of retirement wealth for retirement investors in the SCF. Since RIs typically have most or all of their investable wealth in retirement accounts (see Appendix A.3.1), this large sample of RIs provides a good overview of how the investable wealth of typical U.S. retail investors is allocated. However, the analysis of investment portfolios misses some of the other sources of wealth that households hold – in particular, housing wealth and bank accounts. Appendix A.3.1 provides statistics on the composition of wealth of RIs in the SCF and discusses robustness of the portfolio results.

### 3.3 Summary Statistics

Table 1 reports summary statistics on the sample of RIs with income observations. The average age is 44, 42% of the sample is female, and 71% is married.<sup>15</sup> Partly due to the selection requirements on consecutive income observations that the empirical strategy requires, investors in the sample are relatively well off in terms of income: the median income is \$79K, and the average income is \$106K. Figure 3 compares the distribution of income to RIs in the SCF, where income is measured at the household level. When restricting both samples to single (unmarried) individuals, it is clear that sample is tilted towards higher-income population. Investors in the sample also tend to have relatively stable jobs: the median tenure is 8.6 years.

The average equity share of investor portfolios is 77%. The remaining portfolio assets are

<sup>15</sup>Appendix Figure A.7 plots the age distribution and shows that the sample is balanced by age.

Table 1: Summary Statistics

Variable	Mean	SD	P10	P25	P50	P75	P90
Age	44.10	8.36	32	37	44	51	56
Female	0.42	0.49	0	0	0	1	1
Married	0.72	0.45	0	0	1	1	1
Income	105,640	153,101	38,457	53,103	79,186	119,691	181,264
Employment tenure (years)	10.48	8.19	2.00	3.87	8.58	15.29	21.94
Investable wealth	112,236	236,763	7,393	16,742	48,347	136,018	296,147
Retirement wealth	105,063	145,973	7,334	16,557	47,523	132,527	285,499
Portfolio shares							
Equity	0.77	0.21	0.54	0.70	0.84	0.89	0.97
Fixed income	0.19	0.18	0.01	0.09	0.14	0.25	0.38
Cash	0.03	0.14	0.00	0.00	0.00	0.00	0.03
Alternative	0.01	0.04	0.00	0.00	0.00	0.00	0.00
Market beta of portfolio	0.80	0.23	0.55	0.73	0.85	0.93	0.99
Market beta of equity	1.00	0.11	0.95	1.00	1.00	1.01	1.06
Default investor	0.43	0.49	0.00	0.00	0.00	1.00	1.00
Share in TDFs	0.53	0.44	0.00	0.00	0.56	1.00	1.00
Share in auto rebalancing funds	0.03	0.13	0.00	0.00	0.00	0.00	0.01
Share in individual stocks	0.05	0.14	0.00	0.00	0.00	0.00	0.16
International share of equity	0.06	0.12	0.00	0.00	0.00	0.07	0.25
Investor-initiated trade	0.22	0.41	0	0	0	0	1
Portfolio turnover	0.13	0.51	0.00	0.00	0.00	0.00	0.30
Months with web login	4.03	4.00	0	0	3	7	11

*Notes:* This table presents summary statistics on demographics, wealth, portfolio allocations, and engagement of the sample of retirement investors as of December 31, 2016.

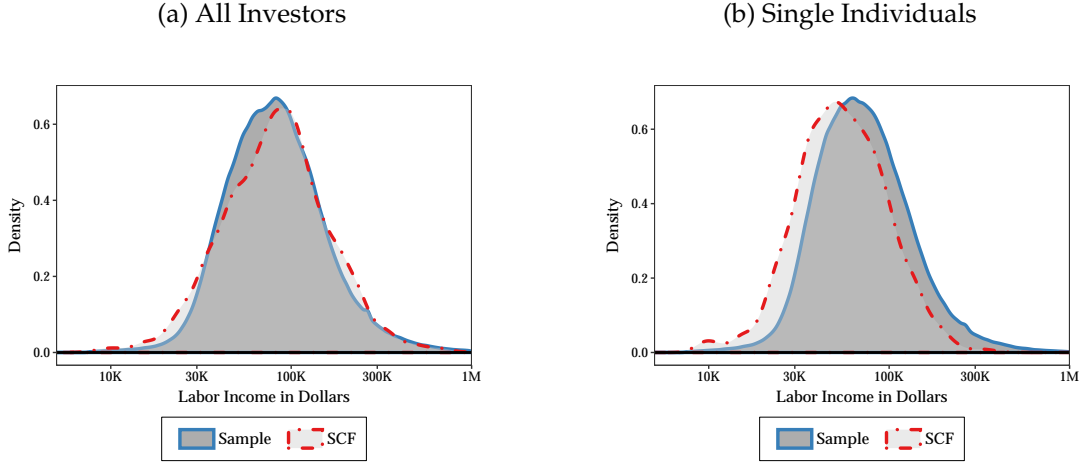
largely invested in fixed income, with an average share of 19%. Only a small fraction of the sample hold cash or other assets. There is substantial heterogeneity in portfolio allocations: the cross-sectional standard deviation of equity shares is 21%. Market betas of investor portfolios tend to be slightly above equity shares, with a dispersion that is similar to the variation in equity shares. Most investors hold an equity portfolio that has a market beta close to one, and bear limited idiosyncratic risk. On average, only 5% of portfolio assets are invested in individual stocks.

Since retirement accounts form an important part of the sample, many investors have significant allocations to target date funds (TDFs). The average TDF share of portfolio assets in the sample is 53%. An important reason for the prominence of TDFs is default allocations. For almost all investors, the default allocation is a TDF. I define a default investor to be someone who has 100% of assets invested in the default fund in an employment-based account, or at least 90% in TDFs in a personal retirement account and the remainder in cash. Default investors cover 43% of the sample.

Many investors are not actively engaged with their portfolios. I define an investor-initiated trade to be a trade or exchange that is not associated with an account inflow or outflow. Just 22% of people in the sample have an investor-initiated trade over the year. As another measure of portfolio rebalancing, I define portfolio turnover as one half times the sum of absolute value of investor trades divided by beginning-of-year financial wealth. The average portfolio turnover is



Figure 3: Income Distribution



Notes: This figure plots the distribution of individual income in the sample of RI investors versus the distribution of household income for RI investors in the SCF. Panel (a) plots the full distribution. Panel (b) restricts the sample to individuals (heads) that are unmarried.

13%. To measure engagement, I calculate in each year the number of months with at least one web login. On average, retirement investors have four months with web visits during the year. There is substantial heterogeneity in engagement: the bottom 10% have no web logins, while the top 10% log in at least once in nearly every month.

## 4 Income and Portfolio Allocations

As emphasized by the stylized model in Section 2, the joint portfolio responses to changes in human capital and to financial wealth are informative about risk preferences. I start the empirical analysis by zooming in on the effects of income growth on changes in equity allocations in investor portfolios – the full estimation of  $b_1$  and  $b_2$  requires a model of rebalancing behavior and is the focus of Section 5. Motivated by the stylized model, the objective here is to estimate the following system of equations:

$$\Delta_h \log \theta_{it} = b_1^p \Delta_h p_{it} + \delta' X_{i,t-h} + \eta_{it} \quad (12a)$$

$$y_{it} = g_{it} + p_{it} + \epsilon_{it} + e_{it}, \quad (12b)$$

where log income  $y$  is decomposed into a predictable component  $g$ , a persistent component  $p$ , a transitory component  $\epsilon$ , and measurement error  $e$ . Assume that predictable income growth is of the form  $\Delta_h g_{it} = \lambda' Z_{i,t-h}$ , where  $Z$  is a vector of deterministic life-cycle characteristics that are included in the controls  $X_{i,t-h}$ .

First, in Section 4.1, I test whether portfolio risk taking varies with overall income in a baseline OLS regression. Second, Section 4.2 makes the distinction between persistent and transitory

income shocks and estimates the effect of persistent income growth on equity share changes. Third, Section 4.3 discusses the effects of portfolio frictions, in particular infrequent rebalancing. Appendix A.3 contains additional details and robustness checks.

## 4.1 Income Growth and Equity Share Changes

First, I analyze the basic relation between income and portfolio allocations. Since cross-sectional variation in income and portfolio compositions is correlated with many sources of heterogeneity across investors, I use panel data to measure the impact of *changes* in income on *changes* in the equity share of investor portfolios.

**Regression setup.** As a first pass, consider an OLS regression of changes in log equity shares,  $\Delta_h \log \theta_{it}$ , on income growth  $\Delta_h y_{it}$ , where  $\Delta_h$  is the  $h$ -year difference operator:

$$\Delta_h \log \theta_{it} = b_1^y \Delta_h y_{it} + \delta' X_{i,t-h} + \eta_{it}. \quad (13)$$

I control for ex-ante differences in demographics and financial profiles by including a basic set of controls  $X_{i,t-h}$ . As illustrated by Appendix Figure A.8 that plots the average change in portfolio equity share as a function of the initial equity share, portfolio shares are mean reverting: low initial equity shares are associated with subsequent increases in equity shares, and vice versa. This effect is partly mechanical, since portfolio shares are bounded between 0 and 1. To account for the observed mean reversion in portfolio shares, I include the initial equity share as a control.

The vector  $X_{i,t-h}$  of controls further includes a second-order polynomial in age, gender, marital status, a second-order polynomial in employment tenure, log income, and the log of financial assets, all measured at  $t - h$ . In addition, I control for fixed effects by year. For the purpose of running OLS regressions, I limit the sample to 90% of the range of wage changes by trimming income growth at the 5th percentile and 95th percentile.<sup>16</sup>

The main outcome variable is the change in log equity share. There are two main reasons for using the logarithm of the equity share. First, if the expected return  $\mu_e$  and volatility  $\sigma_e$  of equity are constant, a basic power utility investor with risk aversion  $\gamma_i$  chooses a constant equity share equal to  $\frac{\mu_e - r_f}{\gamma_i \text{Var}[R^e]}$  (Samuelson, 1969; Merton, 1969). Hence, when taking the difference of the log equity share, we can control for static heterogeneity in beliefs. Second, I assume that the equity share of total financial wealth is a multiple of the measured equity share, with a scaling factor that is independent of wealth fluctuations after controlling for predictable variation in observables.<sup>17</sup>

<sup>16</sup>This trimming reduces the effect of outliers in income growth on the measured portfolio responses, for example due to measurement error from part-year income observations. Section 4.2 formally addresses measurement error.

<sup>17</sup>Note that the data only cover wealth in investment accounts, mainly through retirement accounts. For a large part of the U.S. population, quasi-liquid retirement wealth is the main component of investable wealth. However, this misses other forms of financial wealth, predominantly consisting of bank accounts and other non-equity holdings. Therefore, a reasonable approximation for the typical Retirement Investor is that the equity share of total financial wealth is  $\theta_{it}^f = \phi_{it} \theta_{it}$ , where  $\theta_{it}$  is the observed equity share of investable wealth and  $\phi_{it} \in (0, 1]$  is the share of investable wealth in total financial wealth. I assume that  $\Delta_h \log \phi_{it}$  is independent of wealth changes after controlling for  $X_{i,t-h}$ . Under

Table 2: Income Growth and Equity Share Changes – OLS

	$\Delta_h \log \text{equity share}_t$					
	$h = 1 \text{ year}$			$h = 2$	$h = 3$	$h = 5$
	(1)	(2)	(3)	(4)	(5)	(6)
$\Delta_h \log \text{income}_t$	0.0191 (0.0009)	0.0167 (0.0009)	0.0125 (0.0009)	0.0174 (0.0010)	0.0208 (0.0011)	0.0252 (0.0015)
Year FE	Y	Y				
Log equity share $_{t-h}$		Y	Y	Y	Y	Y
Demographic controls		Y	Y	Y	Y	Y
Employer $\times$ year FE			Y	Y	Y	Y
R-squared	0.022	0.142	0.169	0.229	0.272	0.361
Share of individuals	92.7%	92.7%	92.7%	70.2%	53.4%	33.6%

*Notes:* This table presents the results of an OLS regression of changes in log equity shares on income growth, measured over several horizons. The demographic controls include a second-order polynomial in age, gender, marital status, a second-order polynomial in employment tenure, log income, and the log of financial assets, all measured at  $t - h$ . Standard errors are clustered at the individual level.

**OLS results.** Table 2 presents OLS regression estimates for various sets of controls and horizons. The regressions show a positive and significant relation between income growth and portfolio equity share changes in all specifications. The estimated effect is slightly smaller but still highly significant when including employer–year fixed effects, thereby controlling for any variation at the employer level that may be correlated with changes in asset menus.

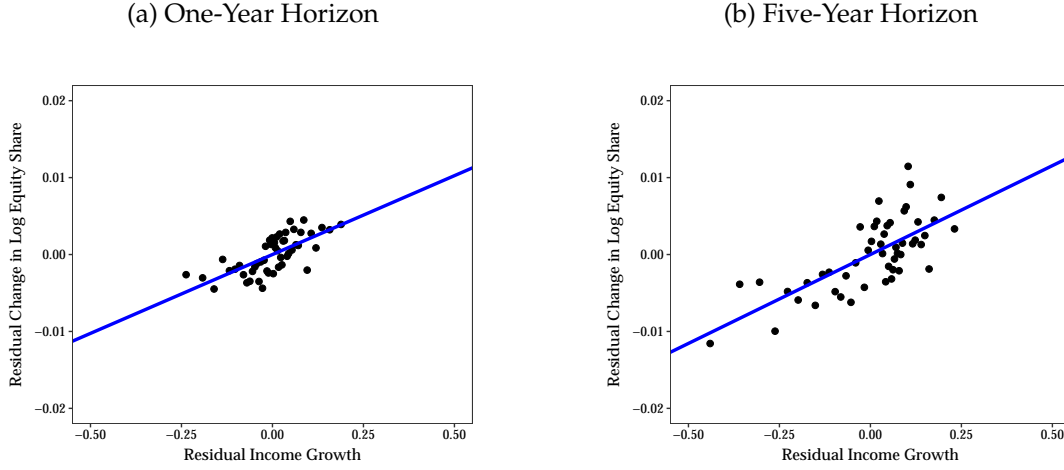
Columns (4)–(6) report the results for longer horizons. The magnitude of the effect increases with the horizon. We would expect the coefficient to increase with the horizon if investors respond more to persistent income growth than to transitory income growth, since the variance of persistent income growth increases with the horizon while the variance of transitory income growth does not. In Section 4.2, I directly measure the effects of persistent income growth.

**Nonlinear effects.** Figure 4 shows a binscatter plot of the relation between income growth and log equity share changes at one-year and five-year horizons, after taking out the effects of demographic differences and ex-ante differences in portfolios. Not surprisingly, the cross-sectional variance of income growth greatly increases with the horizon. The relation between income growth and changes in log equity shares is S-shaped: the elasticity is largest when income growth is relatively close to zero. This variation could either be due to a different sensitivity of portfolios to small versus large shocks in income, or due to the role of different components of income growth: if small income shocks are more likely to be persistent, we would expect to find a larger elasticity of equity shares to income for small income shocks. The results in Appendix A.3.3 suggest that most of the heterogeneity in effect size by different magnitudes of income shocks can be explained by the differences in the responses to persistent and transitory income shocks.

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this assumption, using  $\theta_{it}$  as a proxy for  $\theta_{it}^f$  does not lead to a bias when using the change in logs as outcome variable.

Figure 4: Income Growth and Equity Share Changes



*Notes:* This figure shows binscatter plots that illustrate the OLS regression of changes in log equity shares on income growth, measured over a one-year and five-year horizon. The variables on both axes are orthogonalized with respect to the basic set of demographic controls and the initial log equity share. The demographic controls include a second-order polynomial in age, gender, marital status, a second-order polynomial in employment tenure, log income, and the log of financial assets, all measured at  $t - h$ .

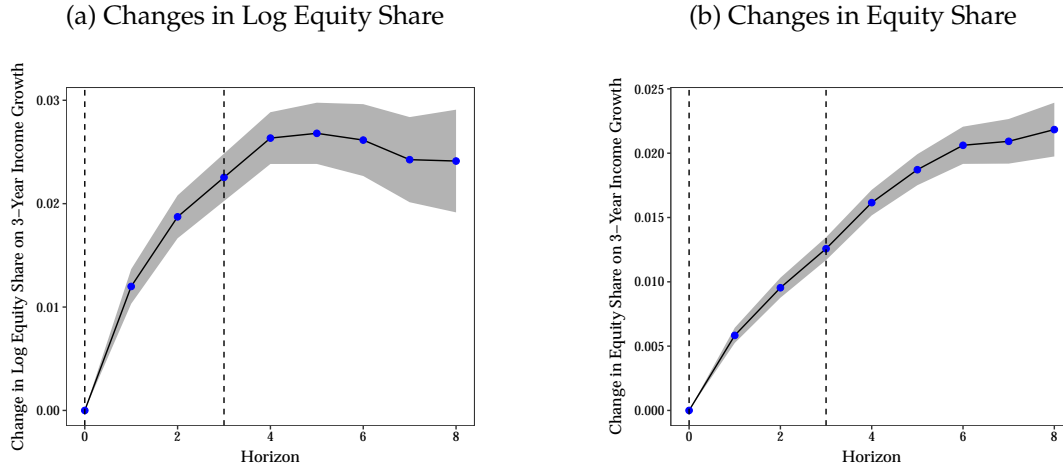
**Long-term effects.** Income growth is positively related to changes in equity shares. A homothetic model with CRRA predicts that there should be an offsetting effect of portfolio returns, but in Section 5 I show that there is no such offsetting effect in the data. The positive relation between income growth and equity share changes is therefore evidence of non-homothetic risk preferences. There are several important models of financial behavior that generate a negative relation between wealth and risk aversion. The stylized model in Section 2 has a subsistence level to generate DRRA. A closely related alternative is the habit model. Habits have similar implications to a subsistence level when they are slow moving, as is typically the case in finance models (e.g. [Campbell and Cochrane, 1999](#)). Another alternative that generates DRRA preferences is consumption commitments ([Chetty and Szeidl, 2007, 2016](#)). Finally, rich households could be consuming a different bundle of consumption goods, with a lower curvature over “luxury” goods. This luxury good may be wealth itself, as in models with a “spirit of capitalism” ([Bakshi and Chen, 1996; Carroll, 2000, 2002](#)).

Distinguishing these different theories is challenging with the available data. However, one important test is whether the portfolio changes are temporary or long lasting. Are the previous findings about temporary changes in risk aversion (e.g. macro habits) or about level effects of wealth on risk preferences? I estimate the effects of income growth on equity shares at future horizons. I estimate the following regression equation at future horizon  $j$ :

$$\Delta_{h+j} \log \theta_{i,t+j} = b_1^j \Delta_h y_{it} + \delta' X_{i,t-h} + \eta_{it}. \quad (14)$$

Figure 5 plots the coefficients of equity share changes over all horizons on three-year income

Figure 5: Long-Run Effects of Three-Year Income Growth



Notes: This figure plots the coefficients of an OLS regression of changes in equity shares, measured over several horizons, on three-year income growth. Panel (a) shows the coefficients when the outcome is the change in log equity shares and panel (b) shows the coefficients when the outcome is the change in the level of the equity share. The regressions control for the initial equity share, a second-order polynomial in age, gender, marital status, a second-order polynomial in employment tenure, log income, and the log of financial assets, all measured at  $t - 3$ . Standard errors are clustered at the individual level.

growth, both when using the equity share in logs and in levels.<sup>18</sup> There is no evidence that the equity share reverts back to the original level. These results suggest that the effects of income on equity shares are long lasting, consistent with a model where risk tolerance depends on the level of wealth, e.g. through a subsistence level or bequest motive.<sup>19</sup> While estimates using log equity shares in panel (a) suggest that the effect of income growth on changes in equity shares does not further increase in magnitude more than one year in the future, the estimated effect of income growth increases monotonically with the horizon when using levels of equity shares in panel (b). This finding is consistent with a model of infrequent portfolio adjustment that is (mostly) driven by external factors, which will be a key focus of the later analysis.

## 4.2 Persistent Income Shocks

Portfolio choice theory predicts a notable difference in the portfolio effects of persistent and transitory income growth. Transitory income shocks have an impact on cash on hand in the current period, but do not change the present value of future labor income. In contrast, persistent income shocks change both the amount of resources available in the current period and total

<sup>18</sup>The corresponding regression estimates are in Appendix Table A.10.

<sup>19</sup>Unless endogeneity in the cross-sectional relation between income and risk taking offsets the positive effect within individual, we would expect a positive relation between level of equity share and level of income. Columns (5)–(6) in Appendix Table A.10 display the results of a cross-sectional regression of the log equity share on log income. Indeed, there is a positive correlation between income and the portfolio equity share even within employer. The estimated elasticity is similar to the estimate from running the regression in first differences. The estimated elasticity is somewhat larger for investors without a default allocation.

human capital. Recall that we are interested in the effect of persistent income growth on portfolios:

$$\Delta_h \log \theta_{it} = b_1^p \Delta_h p_{it} + \delta' X_{i,t-h} + \eta_{it}.$$

I use an IV approach to consistently estimate the effect of persistent income growth on equity shares. In particular, I use lead and lag observations of income to filter out transitory shocks and measurement error (Guiso, Pistaferri, and Schivardi, 2005; Blundell, Pistaferri, and Preston, 2008). To introduce this approach, make the following assumptions: first, all innovations are i.i.d. across investors  $i$ . Second, controls  $X$  are uncorrelated with  $p$ ,  $\epsilon$ ,  $e$ , and  $\eta$ . Third, income measurement error  $e_{it}$  is i.i.d. and uncorrelated with residual portfolio changes  $\eta$ . Fourth, future and past income innovations are uncorrelated with  $\eta_{it}$ :  $\text{Cov}(p_{i,t-h-j}, \eta_{it}) = \text{Cov}(p_{i,t+j}, \eta_{it}) = \text{Cov}(\epsilon_{i,t-h-j}, \eta_{it}) = \text{Cov}(\epsilon_{i,t+j}, \eta_{it}) = 0$  for all  $j > 0$ .

**OLS bias.** If portfolio equity shares respond differently to transitory income shocks than to persistent income shocks or when there is measurement error in income, the OLS regression of equity changes on income growth will give a biased estimate of  $b_1^p$ :

$$b_1^{\text{OLS}} = \frac{\text{Cov}(\Delta_h \log \theta_{it}, \Delta_h y_{it})}{\text{Var}(\Delta_h y_{it})} = b_1^p - \left( b_1^p - \frac{\text{Cov}(\Delta_h \epsilon_{it}, \eta_{it})}{\text{Var}(\Delta_h \epsilon_{it}) + \text{Var}(\Delta_h e_{it})} \right) \frac{\text{Var}(\Delta_h \epsilon_{it}) + \text{Var}(\Delta_h e_{it})}{\text{Var}(\Delta_h y_{it})}. \quad (15)$$

The OLS estimate of the effect of income growth on portfolio allocations suffers from an attenuation bias due to measurement error in income, and is further biased downwards if the portfolio response to transitory income shocks is smaller than to persistent income shocks.

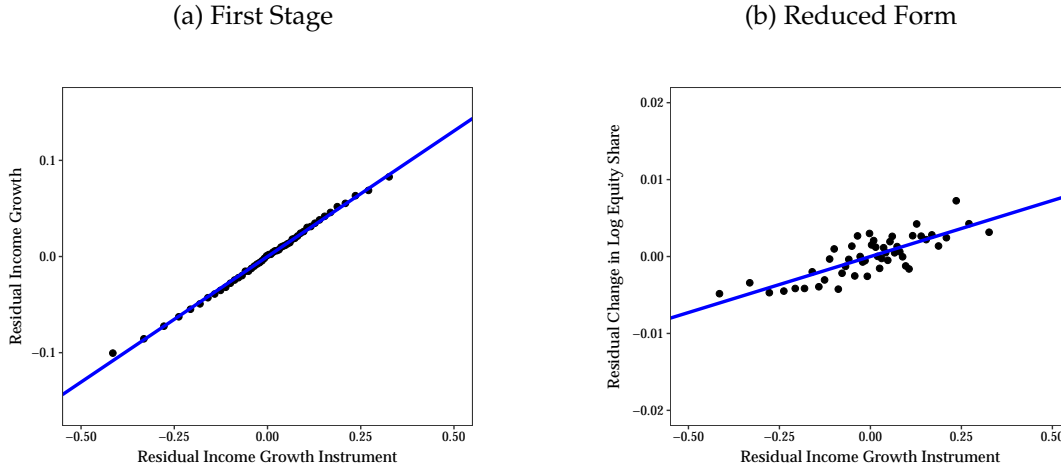
**IV approach.** To address the OLS bias and separate the effects of persistent income shocks from transitory income shocks, I use lead and lag observations of income for identification (Guiso et al., 2005; Blundell et al., 2008). To estimate the effect of persistent shocks, I instrument income growth by income growth over a wider horizon. Specifically, I use the long difference  $\Delta_h^+ y_{it} \equiv y_{i,t+1} - y_{i,t-h-1}$  as an instrument for  $\Delta_h y_{it}$ . Under the above assumptions, this IV approach yields a consistent estimate of  $b_1^p$ :

$$b_1^{\text{IV}} = \frac{\text{Cov}(\Delta_h \log \theta_{it}, \Delta_h^+ y_{it})}{\text{Cov}(\Delta_h y_{it}, \Delta_h^+ y_{it})} = b_1^p \frac{\text{Cov}(\Delta_h p_{it}, \Delta_h^+ p_{it})}{\text{Cov}(\Delta_h p_{it}, \Delta_h^+ p_{it})} = b_1^p. \quad (16)$$

The fourth assumption requires that past income is uncorrelated with current portfolio changes. Since portfolio adjustment is sluggish, this assumption is likely to be violated. Therefore, I add the lag of log financial wealth and the lag of log income to the controls  $X$ , where  $y_{i,t-1}$  is instrumented by  $y_{i,t-2}$ . I assume that these controls appropriately capture the effects of past income growth on current portfolio changes.

Figure 6a shows a binscatter plot for the first stage, using the same set of controls  $X_{i,t-h}$  as before. Figure 6b shows a binscatter plot for the reduced form. Like the baseline OLS specification,

Figure 6: Income Growth and Equity Share Changes, IV Specification



*Notes:* This figure shows binscatter plots that illustrates the IV regression of changes in log equity shares on income growth. Panel (a) illustrates the first stage and panel (b) illustrates the reduced-form specification. The variables on both axes are orthogonalized with respect to the basic set of demographic controls and the initial log equity share. The demographic controls include a second-order polynomial in age, gender, marital status, a second-order polynomial in employment tenure, log income, and the log of financial assets, all measured at  $t - 1$ .

the reduced-form scatter plot shows a positive relation between income growth and changes in log equity shares. As expected when there are no anticipation effects, the reduced-form specification has a flatter slope than the OLS specification.

**IV results.** Table 3 presents the results for IV regressions of log equity share changes on income growth, where income growth is instrumented by the long difference  $\Delta^+ y_{it}$  of log income. Column (1) shows the estimate with time fixed effects as the only control. The specification in column (2) includes the basic set of demographic controls, the initial log equity share, and the interaction of demographic controls with the initial log equity share. The estimated coefficient implies that a 10% income growth leads to an increase in equity share of 0.42%.

The remaining columns in Table 3 show that the results are robust to including various additional sets of controls. In column (3), I interact all controls by yearly dummies to account for possible heterogeneity over time, for instance due to varying market conditions. Column (4) includes zip code by year fixed effects and shows that the relation between income growth and equity share changes is not driven by geographical variation.

The final sets of controls highlight that the results are driven by the effect of individual changes, not by changes at the employer level. Employer-driven changes are a possible confounding factor, for example through employer actions in retirement portfolios or due to the effect of employer-wide changes on investment opportunities. In column (5), I include 3-digit NAICS fixed effects interacted by yearly dummies to control for employment industry effects. Column (6) controls for employer-year fixed effects, and, ultimately, column (7) has employer by income bin by year fixed



Table 3: Income Growth and Equity Share Changes – IV

	$\Delta \log \text{equity share}_t$						
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
$\Delta \log \text{income}_t$	0.0404 (0.0014)	0.0417 (0.0013)	0.0433 (0.0013)	0.0409 (0.0014)	0.0428 (0.0013)	0.0428 (0.0014)	0.0482 (0.0016)
Year FE	Y	Y	Y				
Log equity share $_{t-1}$		Y		Y	Y	Y	Y
Demographic controls		Y		Y	Y	Y	Y
Log equity share $_{t-1} \times \text{year FE}$			Y				
Demographic controls $\times \text{year FE}$			Y				
Zip code $\times \text{year FE}$				Y			
Industry $\times \text{year FE}$					Y		
Employer $\times \text{year FE}$						Y	
Employer $\times \text{income bin} \times \text{year FE}$							Y
Income instrumented	Y	Y	Y	Y	Y	Y	Y
R-squared	0.021	0.144	0.155	0.165	0.147	0.170	0.182
Share of individuals	96.7%	96.7%	96.7%	93.5%	96.3%	96.7%	96.4%

*Notes:* This table presents the results of an IV regression of one-year changes in log equity shares on income growth. The demographic controls include a second-order polynomial in age, gender, marital status, a second-order polynomial in employment tenure, log income, and the log of financial assets, all measured at  $t - 1$ . Log income at  $t - 1$  and  $t$  is instrumented by log income at  $t - 2$  and  $t + 1$ , respectively. Standard errors are clustered at the individual level.

effects. The coefficients in these specifications are not meaningfully different from the baseline estimate.

### 4.3 Portfolio Frictions

The empirical results so far have shown a positive relation between income growth and changes in portfolio equity shares. Although this relation is highly statistically significant, the economic magnitude of the effect is modest, even when measuring the effect of persistent income growth that is more than double the effect of overall income growth. The relatively low effect size could either be evidence of a weak overall relation between income and desired portfolio allocations, or could arise from frictions that limit the transmission from risk preferences to portfolios. I consider three such frictions: infrequent portfolio adjustment, tax implications, and portfolio bounds.

**Infrequent portfolio adjustment.** The main portfolio friction that constrains portfolio choice is infrequent portfolio rebalancing. I find that the average effect of income growth on equity share changes is driven by a relatively small number of investors that make significant changes to their portfolios. It is well known that many investors are passive in selecting and rebalancing their portfolios, in particular among retirement accounts (e.g. [Madrian and Shea, 2001](#); [Agnew, Balduzzi, and Sunden, 2003](#)). As a result, not all changes in desired portfolio allocations are reflected by realized portfolio rebalancing.

Table 4: Income Growth and Equity Share Changes – Alternative Specifications

	$\Delta \log \text{ equity share}_t$				$\Delta \text{ equity share}_t$		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
$\Delta \log \text{ income}_t$	0.0385 (0.0013)	0.0397 (0.0013)	0.0494 (0.0042)	0.0300 (0.0068)	0.0229 (0.0005)	0.0227 (0.0005)	0.0211 (0.0005)
Year FE	Y	Y	Y	Y	Y	Y	Y
Portfolio measure $_{t-1}$	Y	Y	Y	Y	Y	Y	Y
Demographic controls	Y	Y	Y	Y	Y	Y	Y
Income instrumented	Y	Y	Y	Y	Y	Y	Y
Sample / measure	Price-constant portfolios	Retirement	Retirement, non-ret owners	Non-retirement	All	Interior equity share	Price-constant portfolios
R-squared	0.124	0.144	0.177	0.125	0.121	0.138	0.091
Share of individuals	97.1%	96.6%	12.8%	7.7%	100.0%	91.5%	99.9%

*Notes:* This table presents the results of an IV regression of one-year changes in log equity shares on income growth for various subsamples and alternative equity share measures. Column (1) has the price-constant portfolio log equity share change as outcome variable. These hypothetical price-constant portfolios are constructed by starting from beginning-of-period asset holdings, assuming that there are no price changes, and adding to these holdings all inflows and outflows at the asset level. The price-constant log equity share change is the difference between the log equity share of the price-constant portfolio and the initial log equity share. In columns (2) and (3), the outcome variable is the log equity share change of retirement assets. Column (3) limits the sample to non-retirement account owners. Column (4) has the log equity share change of non-retirement assets as outcome variable. Columns (5) and (6) focus on the change in the level of the equity share, where column (6) restricts the sample to interior initial equity shares. Column (7) has the price-constant equity share change as outcome variable. The demographic controls include a second-order polynomial in age, gender, marital status, a second-order polynomial in employment tenure, log income, and the log of financial assets, all measured at  $t - 1$ . Log income at  $t - 1$  and  $t$  is instrumented by log income at  $t - 2$  and  $t + 1$ , respectively. Standard errors are clustered at the individual level.

First, I establish that the elasticity of equity shares to income is in fact driven by investor reallocation decisions. Infrequent portfolio rebalancing means that portfolio allocations are sensitive to fluctuations in asset prices. While the portfolio regressions control for aggregate effects, a correlation between individual income growth and idiosyncratic portfolio returns combined with infrequent portfolio rebalancing could create a spurious relation between income growth and observed equity shares. I therefore track hypothetical price-constant portfolios that are constructed as if there were no changes in valuations, by adding security-level trades to positions at the beginning of the year. I continue to use the IV specification that was introduced in the previous subsection. Columns (1) and (7) of Table 4 illustrate that the estimated effect of income growth on equity shares of price-constant portfolios almost fully captures the effect on overall portfolios.

Second, I find that the responses to income growth are bigger for investors that do not hold the default retirement fund allocation. A robust and important feature of retirement portfolio choice is that default allocations are very sticky. [Choi, Laibson, Madrian, and Metrick \(2004\)](#) find that more

Table 5: Income Growth and Equity Share Changes – Heterogeneity in Portfolio Adjustment

	$\Delta \log \text{equity share}_t$		
	(1)	(2)	(3)
$\Delta \log \text{income}_t$	0.0260		
$\times \text{Default investor}$	(0.0015)		
$\Delta \log \text{income}_t$	0.0495		
$\times \text{Non-default investor}$	(0.0021)		
$\Delta \log \text{income}_t$		-0.0033	
$\times \text{Turnover}_t = 0$		(0.0009)	
$\Delta \log \text{income}_t$		0.0642	
$\times \text{Turnover}_t \in (0, 25\%]$		(0.0036)	
$\Delta \log \text{income}_t$		0.2955	
$\times \text{Turnover}_t > 25\%$		(0.0081)	
$\Delta \log \text{income}_t$			0.0087
$\times \text{Months with web visits}_t \leq 1$			(0.0013)
$\Delta \log \text{income}_t$			0.0499
$\times \text{Months with web visits}_t \in [2, 6]$			(0.0023)
$\Delta \log \text{income}_t$			0.0809
$\times \text{Months with web visits}_t \geq 7$			(0.0034)
Year FE	Y	Y	Y
Log equity share $_{t-1}$	Y	Y	Y
Demographic controls	Y	Y	Y
Income instrumented	Y	Y	Y
R-squared	0.155	0.267	0.166
Share of individuals	82.3%	96.7%	96.7%

*Notes:* This table presents the results of an IV regression of one-year changes in log equity shares on income growth interacted by various indicators. The demographic controls include a second-order polynomial in age, gender, marital status, a second-order polynomial in employment tenure, log income, and the log of financial assets, all measured at  $t - 1$ . Log income at  $t - 1$  and  $t$  is instrumented by log income at  $t - 2$  and  $t + 1$ , respectively. Standard errors are clustered at the individual level.

than half of automatically enrolled participants stick with the default allocation even after three years. Since investors with default allocations have never made an active decision to rebalance, we would expect the elasticity of equity shares to income growth to be smaller for this sample. Column (1) of Table 5 confirms that the coefficient on income changes is nearly twice as large for investors without a default allocation.

Third, I show that portfolio responses to persistent income growth are substantially higher for people that actively rebalance their portfolios. Recall from the summary statistics in Section 3.3 that trading activity and portfolio turnover is low across the sample. This means that even for investors that have opted out of the default allocation or have chosen an initial allocation in a personal account, portfolio reallocation is infrequent. To estimate the effect of income growth on equity shares for the subsample of investors that rebalance their portfolios, I split the set of

investors in three groups based on portfolio turnover over the year. Column (2) of Table 5 shows that the portfolio responses to income growth are strongly increasing in portfolio turnover. The elasticity for those investors that have no portfolio turnover based on their own actions is zero. The effect of income growth on equity share changes is roughly 50% larger than the baseline estimate for investors with some portfolio turnover (below 25%), and the elasticity is as high as 0.3 for investors with a portfolio turnover above 25% (denoted by “active turnover”). These estimates suggest that infrequent portfolio rebalancing is a key driver behind the low sensitivity of portfolio allocations to wealth changes. Section 5 quantifies these effects.

Fourth, I argue that limited attention is a likely explanation for the low average elasticity of portfolio shares due to infrequent trading. I split the sample of investors by the number of months with at least one web login over the year. Column (3) shows that the elasticity of equity shares is strongly increasing in the number of months with web activity. Investors that frequently log into their account adjust their portfolios on average by almost ten times as much as investors that rarely log into their account. I therefore conclude that limited attention is likely to be an important driver behind the small average effect of income growth on equity share changes.

**Tax burden.** The second portfolio friction I consider is limited portfolio adjustment due to tax implications. When capital gains are taxable, investors may be reluctant to rebalance their portfolios in response to wealth shocks in order to avoid realizing a capital gain. The sample of investment accounts consists of both taxable and non-taxable accounts. Retirement accounts are not subject to capital gains and dividend tax, while individual non-retirement accounts are taxable. I examine heterogeneity in the elasticity of equity shares to income by account type. In column (2) of Table 4, the sample is restricted to retirement accounts. The estimated effect is similar to the full sample. Columns (3) and (4) report results for the subsample of investors with non-retirement accounts. Column (3) shows that the estimated effect in retirement accounts is larger for this type of investors, that may be more active in monitoring their portfolios. Column (4) reports a lower coefficient for the same subsample in their non-retirement accounts. This finding suggests that tax considerations play some role in limiting portfolio adjustment in taxable accounts.

**Corner solutions.** Finally, since retail investors generally do not have access to a margin account and cannot short assets, portfolio shares are bounded by 0 and 1. This raises two potential concerns on the analysis of portfolio changes. First, running the regression in logs requires a strictly positive equity share. This restriction drops 4.5% of the sample that hold no equity. If very risk averse investors in fact choose an allocation of zero, the measured effect of income growth might be downward biased. Second, the boundaries of 0% and 100% may lower the sensitivity of equity shares to income changes because the least and most risk-averse investors cannot take more extreme positions when they get hit with a shock. Column (5) of Table 4 reports that the positive relation between income growth and equity share changes carries over to the specification

in levels rather than logs. Column (6) shows that the results are almost identical when restricting the sample to interior equity shares in  $(0, 1)$ . Thus, I conclude that the results are not sensitive to portfolio boundaries.

## 5 Market-Driven Portfolio Fluctuations and Rebalancing Behavior

In this section, I address two key points that arise from the analysis in Section 4. First, many people only rebalance their portfolios infrequently. For most individuals in the U.S., retirement wealth is the main form of investable wealth. Retirement wealth is notoriously “sleepy”, as is well known from the literature on retirement investment. Only 20–25% of households in the full sample of RIs reallocate money across assets in a given year. This means that the measured relation between wealth shocks and portfolio changes likely understates the impact of wealth fluctuations on desired portfolios.

Second, recall from the stylized model that only the combined portfolio responses to income growth and financial returns are informative about non-homotheticity in risk preferences. Infrequent rebalancing induces a mechanical correlation between portfolio returns and portfolio allocation changes. To estimate the effects of portfolio returns on desired allocations, it is therefore crucial to account for fluctuations in portfolios due to irregular portfolio adjustment.

### 5.1 Hazard Rate of Trading

A relatively low percentage of the sample update their portfolios in any given year. The effects of changes in investor financial profiles on risk preferences need to be inferred from this subsample of investors that make an active reallocation decision. For dealing with selection issues and for modeling rebalancing behavior, it is therefore important to understand who trades, and why.

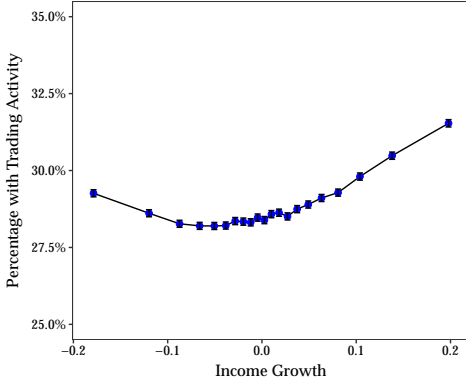
Figure 7a plots the probability of having at least one investor-driven trade over the year as a function of income growth in that year, using 20 bins for income growth. Figure 7b repeats this analysis on the probability of active portfolio turnover (more than 25% of initial assets over the year).<sup>20</sup> Both plots show a hazard rate of rebalancing activity that is nearly flat in income growth. Hence, while the earlier empirical analysis highlighted that income growth affects the *intensive* margin of trading, there is little effect on the *extensive* margin. Appendix Table A.8 presents estimates of an OLS regression of rebalancing activity on the magnitude of income growth and portfolio equity returns. While there is a clear statistical relationship between the magnitude of these shocks and the probability of portfolio adjustment, the incremental R-squared of both income growth and equity returns is nearly zero and the economic magnitudes are limited. For example, an additional 10% positive or negative income growth only increase the probability of trading in the current year by 0.3 percentage points.

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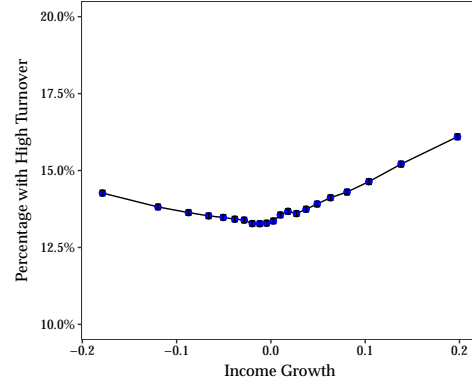
<sup>20</sup>These plots are constructed from the same subsample that is used in the remainder of this section, as described in 5.2. Investors in this subsample have a somewhat higher trading intensity than in the full sample.

Figure 7: Income Growth and Rebalancing Activity

(a) Share of Sample with Trading Activity



(b) Share of Sample with High Turnover



Notes: Panel (a) plots the probability of having an investor-driven trade during the year as a function of income growth in that year, using 20 bins for income growth. Panel (b) plots the probability of having a high portfolio turnover (at least 25% of initial assets) as a function of income growth, again using 20 bins for income growth.

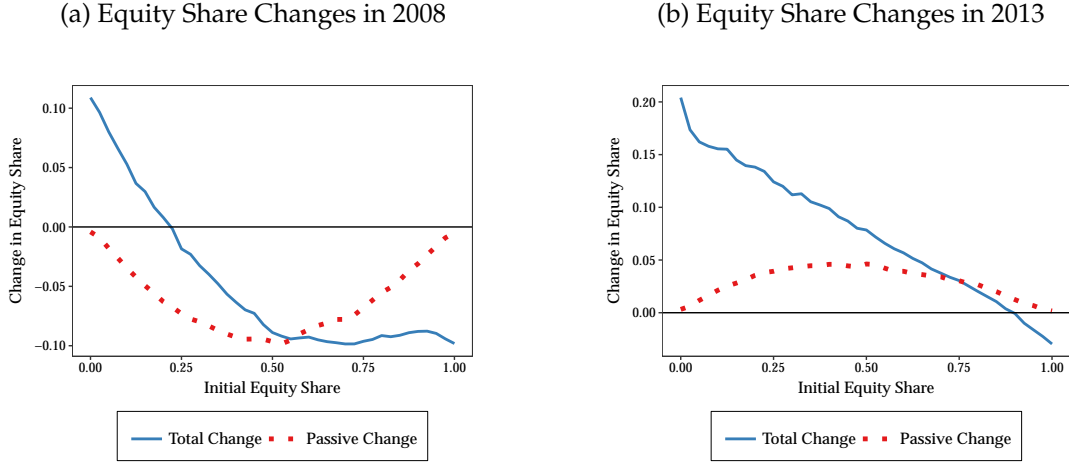
I conclude that trading activity is largely consistent with a model of random, time-dependent adjustment where rebalancing does not vary (much) with changes in observable factors. [Giglio et al. \(2019\)](#) arrive at a similar conclusion in the context of subjective beliefs. Similarly, [Meeuwis et al. \(2019\)](#) find that a small share of investors make large portfolio adjustments in line with their political affiliation in response to the outcome of the 2016 U.S. national election.

## 5.2 Decomposition of Portfolio Changes

**Definition of passive equity share.** When portfolios are not continuously rebalanced, changes in asset allocations are driven both by investor reallocation actions and by realized asset returns. Due to asset return-driven fluctuations, portfolio equity shares will typically rise under good equity market conditions, and decline in a bear market. In order to decompose portfolio changes into appreciation-driven changes and investor reallocation decisions, I calculate the passive equity share of investor portfolios. The passive equity share is defined as the equity share at the end of the year in the absence of any trades during the year. Let  $\omega_{i,j,t-1}$  be the portfolio holdings of investor  $i$  in asset  $j$  at the beginning of year  $t$ . It is straightforward to calculate the passive portfolio weight  $\omega_{ijt}^p$  on security  $j$  from the realized gross return  $R_{jt}$  in year  $t$ :  $\omega_{ijt}^p \equiv \omega_{i,j,t-1} R_{jt}$ . The passive equity share  $\theta_{it}^p$  can then be computed as the equity share of a portfolio with weights  $\omega_{ijt}^p$ .

The passive equity share change  $\Delta^p \theta_{it}$  is the change in equity share over year  $t$  if the investor does not make any trades during that year:  $\Delta^p \theta_{it} \equiv \theta_{it}^p - \theta_{i,t-1}$ . The passive equity change is zero for investors that start with a portfolio that is either 100% or 0% invested in equity. Since the market component captures a large part of the variation in returns on risky assets, the average passive change as a function of initial equity share is inverse U-shaped in years with positive

Figure 8: Decomposition of Portfolio Changes



Notes: This figure plots the average total change in portfolio equity share and the average passive change in portfolio equity share as a function of the initial equity share. Panel (a) shows this decomposition for 2008 (a bear market) and panel (b) shows this decomposition for 2013 (a bull market).

market returns, and U-shaped in years with negative market returns. For illustration, Figure 8 plots the average total change and passive change in equity shares as a function of initial equity share, in a bear market (2008) and bull market (2013). The residual change  $\theta_{it} - \theta_{it}^p$  in equity shares is the change in the portfolio that does not mechanically follow from realized returns and is due to rebalancing. Consistent with the earlier analysis in Section 4, I will run the main empirical specifications in logs. The log passive change in equity shares is analogously defined by  $\Delta^p \log \theta_{it} \equiv \log \theta_{it}^p - \log \theta_{i,t-1}$ .

**Sample.** The variation that passive portfolio changes due to fluctuations in asset prices induce on investor portfolios provides an opportunity to quantify the extent of portfolio rebalancing. Necessary for this decomposition is that investors hold a portfolio that is sensitive to asset fluctuations. I therefore restrict the sample in this part of the analysis to investors with beginning-of-period holdings that generate variation in passive changes to equity shares. In particular, I restrict the sample to investors with an initial equity share between 0.01 and 0.99, and that have less than 100% invested in blended funds that automatically rebalance their asset mix.<sup>21</sup> The resulting sample is likely to be more active in rebalancing their portfolios than the excluded investors for whom we do not observe variation in passive equity changes. Recall from the results in Section 4.3 that there is a consistent but lower effect of income growth on equity shares for investors with a default allocation (usually a TDF), likely because this subset is less engaged with their portfolios.

<sup>21</sup>Note that rebalancing of equity shares in response to market fluctuations is done both by individual investors and by the fund managers of multi-asset class funds. For the purpose of this analysis, I take the perspective of an individual investor by treating multi-asset funds as one asset position like any other fund.



Table 6: Adjustment Model Regression Coefficients

	$\Delta \log \text{equity share}_t$						
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Passive change in log equity share <sub>t</sub>	0.8284 (0.0057)	0.8372 (0.0059)	0.9105 (0.0087)	0.7646 (0.0080)	0.6998 (0.0088)	0.7083 (0.0120)	0.6841 (0.0128)
$\Delta \log \text{income}_t$		0.0525 (0.0021)	0.0513 (0.0023)	0.0576 (0.0044)	0.0619 (0.0039)	0.0617 (0.0045)	0.0648 (0.0079)
Log portfolio return <sub>t</sub>		-0.0095 (0.0023)	-0.0486 (0.0029)	0.0459 (0.0039)	0.0209 (0.0029)	-0.0065 (0.0036)	0.0749 (0.0050)
Year FE	Y	Y	Y	Y	Y	Y	Y
Log equity share <sub>t-1</sub> (3rd order) × year FE	Y	Y	Y	Y	Y	Y	Y
Demographic controls	Y	Y	Y	Y	Y	Y	Y
Income instrumented		Y	Y	Y	Y	Y	Y
Sample			Good market	Bad market	Indiv stocks	Indiv stocks, good market	Indiv stocks, bad market
R-squared	0.163	0.163	0.187	0.088	0.111	0.121	0.069
Share of individuals	56.3%	56.2%	51.0%	39.8%	21.2%	19.3%	14.5%

*Notes:* This table presents regression estimates of the partial adjustment model. The demographic controls include a second-order polynomial in age, gender, marital status, a second-order polynomial in employment tenure, log income, and the log of financial assets, all measured at  $t - 1$ . Log income at  $t - 1$  and  $t$  is instrumented by log income at  $t - 2$  and  $t + 1$ , respectively. The good market years are {2009, 2010, 2012, 2013, 2014, 2016, 2017}; the bad market years are {2008, 2011, 2015}. Columns (5)–(7) restrict the sample to investors that hold individual stocks in their portfolios at  $t - 1$ . Standard errors are clustered at the individual level.

**Baseline rebalancing regression.** When portfolios are infrequently rebalanced, passive changes in equity shares due to market fluctuations have an effect on overall portfolio changes. To examine the relation between passive changes and total changes in equity shares, I run the following rebalancing regression:

$$\Delta \log \theta_{it} = b_0 \Delta^p \log \theta_{it} + \delta' X_{i,t-1} + \eta_{it}. \quad (17)$$

The coefficient  $b_0$  is inversely related to the speed of portfolio adjustment: with full adjustment,  $b_0 = 0$ . When adjustment is partial, we should find  $b_0$  to be between 0 and 1. I run the regression (17) with controls for log initial equity share and the same set of basic demographic characteristics as before.<sup>22</sup> Column (1) of Table 6 reports that the coefficient  $b_0$  on the passive portfolio change is 0.83. This translates to a speed of adjustment of 17% at an annual basis.<sup>23</sup>

<sup>22</sup>Running an OLS regression is problematic here due to measurement error in the equity share of multi-asset class funds that introduces a correlation between  $\Delta^p \log \theta_{it}$  and  $\eta_{it}$ . Therefore, I instrument passive changes by the counterfactual passive change where the equity share of multi-asset class funds is set to the beginning-of-year value. This construction has little effect on the estimation results.

<sup>23</sup>The estimated speed of adjustment is considerably lower than the adjustment speed of 64% in Calvet et al. (2009). This difference is likely caused by sampling differences – Retirement Investors may be more passive than the typical individual that holds risky assets outside of retirement accounts.

### 5.3 Partial Adjustment Model

To quantify the effects of wealth shocks on desired portfolio allocations, I estimate a model of partial portfolio adjustment in the spirit of [Calvet et al. \(2009\)](#).

**Setup.** To be able to infer the effects of wealth shocks on desired portfolio allocations from changes in observed equity shares, while accounting for infrequent portfolio rebalancing, I make three assumptions on portfolios and rebalancing behavior.

First, assume that the realized log equity share of the portfolio is a linear combination of the log passive share  $\log \theta_{it}^p$  and the log desired equity share  $\log \theta_{it}^d$ :

$$\log \theta_{it} = (1 - \chi) \log \theta_{it}^p + \chi \log \theta_{it}^d + \eta_{it}, \quad (18)$$

where the residual  $\eta_{it}$  is i.i.d., reflecting idiosyncratic variation in individual investor portfolios. Subtracting the lagged equity share  $\log \theta_{i,t-1}$  from both sides yields

$$\Delta \log \theta_{it} = (1 - \chi) \Delta^p \log \theta_{it} + \chi (\Delta \log \theta_{it}^d + \log \theta_{i,t-1}^d - \log \theta_{i,t-1}) + \eta_{it}. \quad (19)$$

Note that  $\chi$  determines the speed of adjustment towards the target allocation  $\theta_{it}^d$ . When  $\chi = 1$ , the realized log equity share is equal to the log target share plus a residual component. For  $\chi \in (0, 1)$ , changes in portfolios are driven by both market fluctuations and portfolio rebalancing.

Second, assume that the change in log desired equity share is a linear combination of portfolio factors  $D_{it}$  and lagged investor characteristics  $X_{i,t-1}$ :

$$\Delta \log \theta_{it}^d = \lambda'_d D_{it} + \lambda'_{xt} X_{i,t-1}, \quad (20)$$

where  $D_{it}$  and  $X_{i,t-1}$  are independent of  $\eta_{it}$ . The factors in  $D_{it}$  capture innovations to investor financial profiles that lead to changing in desired allocations, such as wealth shocks. Desired allocations can also change based on ex-ante differences in  $X_{i,t-1}$ , such as age, with an effect that may vary by year.

Third, assume that the initial distance from the target is

$$\log \theta_{i,t-1}^d - \log \theta_{i,t-1} = \zeta'_t X_{i,t-1}. \quad (21)$$

This assumption does not require that desired portfolios take a common form across investors, but imposes the less restrictive condition that the distance of the initial portfolio from the targeted equity share is proxied by observables  $X_{i,t-1}$ .

Combining these three assumptions leads to the following reduced-form specification:

$$\Delta \log \theta_{it} = b_0 \Delta^p \log \theta_{it} + b' D_{it} + \delta'_t X_{i,t-1}, \quad (22)$$

where the coefficients of this specification are related to the underlying structure as follows:  $b_0 =$

$$1 - \chi, b = \chi \lambda_d, \delta_t = \chi \lambda_{xt} + \chi \zeta_t.$$

**Adjustment model estimates.** I estimate (22) in the data. The main object of interest is  $\lambda_d$ , which captures the effects of characteristics  $D_{it}$  on desired portfolio allocations. In particular, I specify  $D_{it}$  to include income growth and the portfolio return. Note that  $1 - b_0$  captures the speed of adjustment, and we can recover  $\lambda_d$  from the estimated regression coefficients via  $b/(1 - b_0)$ .

Table 6 presents the regression estimates. In the baseline estimation of the adjustment model in column (2), the controls include the basic set of demographics characteristics, year fixed effects, and a third-order polynomial in beginning-of-year log equity share interacted by yearly dummies to take out the systematic component of returns and portfolio changes. In this specification, the coefficient of the total change in log equity shares on the passive change is 0.837. Similar to the results from Section 4, the estimated coefficient on (persistent) income growth is 0.053. This estimate translates into a structural effect on changes in the desired log equity share of  $0.053/(1 - 0.837) = 0.323$ . After controlling for the mechanical relation between returns and portfolio allocations due to infrequent rebalancing, I find a negative effect of portfolio returns on changes in the log equity share, with a point estimate of  $-0.010$ . This estimate translates into a structural effect of  $-0.010/(1 - 0.837) = -0.058$  on desired portfolios. The estimated effect of portfolio returns is substantially closer to zero than the income growth effect. As a result, the combined effect is that  $b_1 + b_2 > 0$ , which is evidence of non-homothetic risk preferences.

Next, I examine the variation in estimates by market conditions. Recall that the sample period of portfolio changes is 2007–2017 and therefore spans a range of market conditions. I split the sample in years with a good market return – above the historical average – and years with a bad market return. The good market years are {2009, 2010, 2012, 2013, 2014, 2016, 2017}; the bad market years are {2008, 2011, 2015}. Columns (3) and (4) present the estimation results for these two samples. The estimated effect of income is stable across market conditions. In contrast, the coefficient on idiosyncratic portfolio returns varies considerably by sample. Consistent with the sample in Calvet et al. (2009), the effect of portfolio returns is positive in years with a bad market return. However, we get an opposite effect in years with a good market return. Pooled across all market conditions, the overall result is a small negative effect of portfolio returns on equity shares. Columns (5)–(7) show that the results are similar for the subsample of investors that hold individual stocks and therefore have more idiosyncratic variation in portfolio returns.

## 6 Life-Cycle Portfolio Choice Model

In this section, I present a discrete time life-cycle model that can account for the empirical features of portfolio choice. The model builds on workhorse models of saving and portfolio choice (Cocco et al., 2005) and has three non-standard features: non-homothetic risk preferences, a distribution of idiosyncratic labor income shocks that incorporates countercyclical tail risk, and infrequent portfolio adjustment.

## 6.1 Preferences

Households have finite lives and live from model age  $t = 1$  (actual age 23) to a maximum age of  $T_D = 78$  (actual age 100). The life cycle consists of a working phase and a retirement phase. Households work until age  $T_R = 43$  (actual age 65), after which they retire. The probability that the household survives until the next year conditional on being alive at age  $t$  is denoted by  $\pi_t$ . I set the survival probability during the working phase to one. The age-dependent probabilities of survival during retirement are obtained from U.S. mortality tables.

Households have Epstein-Zin utility over a single consumption good with constant elasticity of intertemporal substitution (EIS)  $\psi$ :

$$V_{it} = \left\{ (1 - \beta_i) C_{it}^{1 - \frac{1}{\psi}} + \beta_i \left( J^{-1} \left( \mathbb{E}_t [\pi_t J(V_{i,t+1}) + (1 - \pi_t) J(W_{i,t+1}/\tilde{b})] \right) \right)^{1 - \frac{1}{\psi}} \right\}^{\frac{1}{1 - \frac{1}{\psi}}}, \quad (23)$$

where  $C_{it}$  is consumption of household  $i$  in year  $t$ ,  $\tilde{b}$  captures the strength of the bequest motive,  $W_{it}$  is cash on hand at beginning of period, and  $J$  is a certainty equivalent aggregation function.

**Risk preferences.** The typical choice for  $J$  in an Epstein-Zin framework is power utility:  $J(v) = \frac{v^{1-\gamma_0}-1}{1-\gamma_0}$ . In this form, relative risk aversion is constant and equal to  $\gamma_0$ . I consider a generalized case of non-homothetic risk preferences, where  $J$  is defined by the ODE<sup>24</sup>

$$-\frac{J''(v; \beta) \cdot v}{J'(v; \beta)} = \gamma_0 (v/\kappa_\beta)^{-\gamma_1}. \quad (24)$$

Note that  $J$  takes the form of power utility when  $\gamma_1 = 0$ . The case with  $\gamma_1 > 0$  is a reduced-form way to capture cross-sectional and time series variation in risk aversion. The coefficient of relative risk aversion decreases as the value function that enters the certainty equivalent calculation increases. Richer households with higher lifetime expected utility therefore have a lower risk aversion. Such a relation can be generated in a multiple good setting with non-homotheticities across goods, e.g. basic versus luxury goods (Wachter and Yogo, 2010) or consumption versus bequests (Carroll, 2000, 2002). Dew-Becker (2014) considers a similar modification of standard Epstein-Zin preferences by choosing a habit-formation utility form for  $J$ .

The purpose of the scaling factor  $\kappa_\beta$  in (24) is to normalize risk aversion and eliminate mechanical heterogeneity in risk aversion through differences in the rate of time preference across investors. I choose  $\kappa_\beta$  to be proportional to the value function of a benchmark investor in the model with fixed CRRA preferences and discount rate  $\beta$ , and such that average risk aversion is approximately equal to  $\gamma_0$  in model simulations.<sup>25</sup>

<sup>24</sup>This ODE does not have a closed-form solution. I solve for  $J$  numerically when solving the life-cycle model. Appendix Figure A.9 plots this parameterization of  $J$  for different values of  $\gamma_1$ .

<sup>25</sup>Note that the scaling factor in (24) varies by the rate of time preference  $\beta$ . As discussed below, I introduce heterogeneity in time discounting across agents  $i$  to match cross-sectional statistics on saving rates. But since local risk aversion depends on the value function, variation in  $\beta$  across individuals leads to variation in risk aversion through the value function. I therefore allow the scaling factor to be  $\beta$ -specific. This approach is conservative by eliminating

**Time discounting.** Since a full exploration of the cross-sectional distribution of preferences is outside the scope of this paper, I assume constant values of the EIS  $\psi$  and risk-aversion parameters  $\gamma_0$  and  $\gamma_1$ . However, I do allow for cross-sectional heterogeneity in time preferences  $\beta_i$ . I assume that discount factors are correlated with initial permanent income. This heterogeneity allows me to capture cross-sectional saving patterns in the data, namely that income-rich households tend to have higher saving rates than poor households (see discussion below in Section 7.1). I use a simple functional form for  $\beta_i$  where the discount rate is a logistic function of initial permanent income. Let  $\beta_i = \beta(P_{i1}; \beta_0, \beta_1)$ , where  $\beta(\cdot)$  is given by

$$\beta(P_1; \beta_0, \beta_1) = 1 - (1 - \beta_0) \frac{2}{1 + P_1^{\beta_1}}, \quad (25)$$

with baseline discount rate  $\beta_0$  and a slope  $\beta_1$  that captures the dependence of the discount factor on initial permanent income. Appendix Figure A.10 plots this parameterization for  $\beta$  as a function of initial permanent income for different values of  $\beta_1$ . When  $\beta_1 = 0$ , the discount rate is constant at  $\beta_0$ . A positive slope parameter  $\beta_1$  implies that discount factors are increasing in initial permanent income  $P_1$ . Discount rates range between  $2\beta_0 - 1$  and 1.

## 6.2 Income

**Working life.** During their working life, households earn wage income that is subject to idiosyncratic risk. Gross income consists of three components: a deterministic age component  $G$ , permanent income  $P$ , and a transitory income shock  $e^\epsilon$ . The process for gross income  $Y_{it}$  is given by

$$\log Y_{it} = \log G_t + \log P_{it} + \epsilon_{it} \quad (26)$$

$$\log P_{it} = \log P_{i,t-1} + \eta_{p,x} x_t + \zeta_{it}, \quad (27)$$

with aggregate permanent shock  $x_t$  and idiosyncratic permanent shock  $\zeta_{it}$ .

Agents pay income taxes over their wage income. I use a parametric form for after-tax income as a function of pre-tax income that captures progressivity in income taxes and is used in the literature by Benabou (2000), Heathcote, Storesletten, and Violante (2017), and others. In particular, after-tax income is given by  $Y_{it}^{post} = (1 - \tau)Y_{it}^{1-\rho}$ . Taxes are progressive when  $\rho > 0$  and neutral when  $\rho = 0$ .

**Retirement.** In retirement, agents receive Social Security benefits. These payments are modeled according to the formulas of the Social Security's Old-Age, Survivors, and Disability Insurance program. Retired households receive a percentage of the economy-wide average wage (equal to \$74K in the SCF and normalized to 1) based on their historical average earnings, subject to a cap.

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an additional source of heterogeneity in risk aversion in the cross section that cannot be identified from within-person changes and would further amplify the cross-sectional effects of non-homothetic risk preferences in the model.

Let  $\bar{Y}_{it_R}$  be average (capped) earnings over the working life at retirement:

$$\bar{Y}_{it_R} = \frac{\sum_{t=1}^{T_R} \min(Y_{it}, 1.60)}{T_R}. \quad (28)$$

The schedule for replacement income is given by

$$f^{soc}(\bar{Y}) = 0.9 \min(\bar{Y}, 0.14) + 0.32 \max(\min(\bar{Y}, 0.83) - 0.14, 0) + 0.15 \max(\bar{Y} - 0.83, 0). \quad (29)$$

Since Social Security replacement income is based on average income over the working life, it would be necessary to keep track of the history of labor income to calculate individual retirement benefits. To save one state variable and limit the computational burden, I instead predict agents' Social Security benefits in retirement based on the terminal value of permanent income at retirement.<sup>26</sup>

### 6.3 Asset Markets

Agents can invest in two assets: one-period risk-free bonds and a risky asset. Investment opportunities are constant over time. The risk-free asset pays a fixed gross return  $R^f$ . The risky asset has return  $R_t^e = \bar{R}^e e^{\nu_t}$ , where  $\bar{R}^e$  is the average gross return to equity. Return shocks are given by

$$\nu_t = -\frac{1}{2}\sigma_v^2 + \eta_{v,x}x_t + u_t. \quad (30)$$

The macro risk variable  $x_t$  is i.i.d. normally distributed:  $x_t \sim N(0, \sigma_x^2)$ . The purely financial shock  $u_t$  is also normally distributed:  $u_t \sim N(0, \sigma_u^2)$ . The total variance of log stock market returns is  $\sigma_v^2 = \eta_{v,x}^2 \sigma_x^2 + \sigma_u^2$ .

### 6.4 Wealth Dynamics

Households enter a period with cash on hand  $W_{it}$ , that is composed of financial wealth and labor income. They decide on how much to consume,  $C_{it}$ , and how much to invest in stocks,  $S_{it}$ , and in bonds,  $B_{it}$ . The budget constraint is given by

$$C_{it} + S_{it} + B_{it} = W_{it}. \quad (31)$$

Wealth is accumulated through labor income and returns on asset positions:

$$W_{i,t+1} = S_{it}R_{t+1}^e + B_{it}R^f + Y_{i,t+1}^{post}. \quad (32)$$

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<sup>26</sup>Specifically, I predict the retirement replacement rate by a third-order polynomial in log permanent income. The  $R^2$  of this regression is 96%.

Following common assumptions in the life-cycle literature, borrowing and short selling are not allowed, so that

$$S_{it} \geq 0 \quad (33)$$

$$B_{it} \geq 0. \quad (34)$$

Let  $\theta_{it} \in [0, 1]$  denote the share of the portfolio that is invested in stocks. The budget constraint can be written in terms of  $\theta_{it}$  as

$$W_{i,t+1} = (W_{it} - C_{it})(R^f + \theta_{it}(R_{t+1}^e - R^f)) + Y_{i,t+1}^{post}. \quad (35)$$

To capture infrequent rebalancing in portfolios, I assume that portfolios can only be updated with some probability each year. Let  $\chi$  be the Calvo frequency of portfolio adjustment. Without rebalancing, the portfolio equity share equals the passive equity share that moves with realized asset returns. Hence, the equity share  $\theta_{it}$  is given by

$$\theta_{it} = \begin{cases} \theta_{it}^{pass} & \text{if no portfolio update at } t, \text{ with probability } 1 - \chi \\ \theta_{it}^* & \text{if portfolio update at } t, \text{ with probability } \chi, \end{cases} \quad (36)$$

where the passive portfolio equity share is

$$\theta_{it}^{pass} = \theta_{i,t-1} \frac{R_t^e}{R^f + \theta_{i,t-1}(R_t^e - R^f)}. \quad (37)$$

I assume that agents can freely choose their initial portfolio  $\theta_{i1}$ .

The agent's objective function is to maximize (23) over  $C_{it}$  and  $\theta_{it}^*$  subject to the budget constraint (35), the dynamics for asset returns and labor income, and the process for  $\theta_{it}$ . Since an analytical solution to this problem does not exist, I solve the model through numerical dynamic programming.

## 6.5 Calibration

Table 7 reports the parameters that are fixed or estimated outside of the model.

**Preferences.** I fix the parameter values of the EIS  $\psi$  and the bequest motive  $\bar{b}$  at standard values. First, since investment opportunities are constant over time, it is hard to separately identify the EIS from the rate of time preference.<sup>27</sup> I therefore fix the EIS to a standard value of  $\psi = 0.5$ . Second, because the focus is on pre-retirement behavior, I do not estimate the bequest parameter. I set this

<sup>27</sup>In recent work, Calvet et al. (2019) show that the EIS can be identified in a standard life-cycle model from endogenous variation in expected returns due to life-cycle profiles in equity shares and due to mortality risk. They find a distribution of EIS that is widely dispersed across households.



Table 7: Calibrated Parameters

Parameter	Description	Value	Source / target
<i>Asset returns</i>			
$R^f$	risk-free rate	1.02	literature
$\overline{R^e} - R^f$	equity premium	0.045	literature
$\sigma_v$	equity volatility	0.18	literature
<i>Income process</i>			
$g_0$	income profile, constant	-4.315	
$g_1$	income profile, coefficient on age	0.319	Cocco et al. (2005)
$g_2$	income profile, coefficient on age <sup>2</sup> /10	-0.058	
$g_3$	income profile, coefficient on age <sup>3</sup> /100	0.003	
$\sigma_{\xi,1}$	volatility of center of permanent income shock	0.064	
$\lambda_{\xi,2}$	probability of left tail of permanent income shock	0.032	
$\mu_{\xi,2}$	mean of left tail of permanent income shock	-0.167	
$\sigma_{\xi,2}$	volatility of left tail of permanent income shock	0.334	McKay (2017)
$\lambda_{\xi,3}$	probability of right tail of permanent income shock	0.019	
$\mu_{\xi,3}$	mean of right tail of permanent income shock	0.394	
$\sigma_{\xi,3}$	volatility of right tail of permanent income shock	0.334	
$\sigma_x$	volatility of macro shock	0.124	derived from McKay (2017)
$\eta_{p,x}$	exposure of income growth to macro shock	-0.237	volatility of aggregate income growth
$\eta_{v,x}$	exposure of stock return to macro shock	-0.921	correlation of stock returns and aggregate income growth
$\sigma_{p_1}$	dispersion of initial permanent income	0.812	Gini coefficient of income
$\tau$	baseline income tax	0.3	literature
$\rho$	tax progressivity	0.181	Heathcote et al. (2017)
<i>Preferences</i>			
$\psi$	elasticity of intertemporal substitution	0.5	literature
$\tilde{b}$	bequest motive	2.5	Gomes and Michaelides (2005)

Notes: This table summarizes the parameter values that are fixed or estimated outside of the model.

parameter to  $\tilde{b} = 2.5$ , similar to Gomes and Michaelides (2005). Appendix A.4.3 shows that the model conclusions are robust to alternative choices for these two parameters.

**Asset returns.** I calibrate the moments of asset returns to standard values in the portfolio choice literature. The real risk-free bond return is set to  $R^f = 1.02$  and the equity premium to 4.5% per year. I set the volatility of stock returns to  $\sigma_v = 18\%$ , so that the Sharpe ratio of equity is 0.25.

**Income.** The age profile  $G$  is given by

$$\log G_t = g_0 + g_1 \cdot t + g_2 \cdot t^2/10 + g_3 \cdot t^3/100. \quad (38)$$

I use the estimated income profile of college-educated households from Cocco et al. (2005) that captures the hump-shaped pattern of earnings over the working life. Income is normalized so

that average income across all working agents in the model is equal to one. I set the baseline income tax rate to  $\tau = 0.3$ , and I use the estimated tax progressivity from [Heathcote et al. \(2017\)](#) with a value of  $\rho = 0.181$ .

In the main specification for the distribution of idiosyncratic income shocks, I incorporate countercyclical tail risk in wage income. I use the specification of permanent idiosyncratic income shocks from [McKay \(2017\)](#) that fits recent empirical evidence on the cyclicity of skewness in income growth as reported by [Guvenen et al. \(2014\)](#).<sup>28</sup> Most people have a common earnings change that is drawn from a distribution  $N(\mu_{1,t}, \sigma_{\xi,1}^2)$ . A fraction  $\lambda_{\xi,2}$  of workers receive a large and persistent earnings loss that is drawn from the distribution  $N(\mu_{2,t}, \sigma_{\xi,2}^2)$ . Similarly, a fraction  $\lambda_{\xi,3}$  of workers receive a very positive shock with distribution  $N(\mu_{3,t}, \sigma_{\xi,3}^2)$ . I assume a perfect correlation between aggregate income shocks and time-varying skewness in idiosyncratic shocks. Hence, we get the following setup for persistent shocks:

$$\xi_{it} \sim \begin{cases} N(\mu_{1t}, \sigma_{\xi,1}^2) & \text{with probability } 1 - \lambda_{\xi,2} - \lambda_{\xi,3} \\ N(\mu_{2t}, \sigma_{\xi,2}^2) & \text{with probability } \lambda_{\xi,2} \\ N(\mu_{3t}, \sigma_{\xi,3}^2) & \text{with probability } \lambda_{\xi,3}, \end{cases} \quad (39)$$

where the macro shock  $x_t$  drives the distribution of the tails and idiosyncratic shocks have mean zero:

$$\begin{aligned} \mu_{1t} &= \bar{\mu}_t \\ \mu_{2t} &= \bar{\mu}_t + \mu_2 - x_t \\ \mu_{3t} &= \bar{\mu}_t + \mu_3 - x_t \\ \bar{\mu}_t &= -\lambda_{\xi,2}\mu_2 - \lambda_{\xi,3}\mu_3 + (\lambda_{\xi,2} + \lambda_{\xi,3})x_t. \end{aligned} \quad (40)$$

The skewness process  $x$  estimated by [McKay \(2017\)](#) is persistent. I set the volatility of  $x$  to match the unconditional volatility of the skewness process. The resulting value of  $\sigma_x$  is 0.124. The other parameter values in the distribution of  $\xi_{it}$  and  $\epsilon_{it}$  are directly taken from [McKay \(2017\)](#).

I use the series of average income growth, net of life-cycle effects, from [Guvenen et al. \(2014\)](#) to set the parameters  $\eta_{p,x}$  and  $\eta_{v,x}$ . I choose  $\eta_{p,x}$  to match the volatility of aggregate income growth of 0.029, and I pick  $\eta_{v,x}$  so that the correlation between aggregate income growth and stock returns is 0.635. This aggregate correlation leads to a correlation of individual permanent income growth with stock returns of 0.152, which is very close to the commonly used value of 0.15 as estimated by [Campbell et al. \(2001\)](#). There is no correlation between transitory shocks and equity returns. Finally, let initial permanent income be given by  $\log P_{i1} \sim N(0, \sigma_{p_1}^2)$ . I calibrate the dispersion of initial log permanent income to match the Gini coefficient of income in the SCF in 2016, which is 0.551.

<sup>28</sup>The full model of [McKay \(2017\)](#) also includes displacement risk. I abstract from unemployment in the model and use the distribution of income growth for employed individuals.

**Alternative income process: no tail risk in labor income.** As an alternative specification of idiosyncratic income risk, I consider the traditional setup of the income process from the life-cycle literature with normally distributed shocks:  $\xi_{it} \sim N(0, \sigma_\xi^2)$  and, as before,  $\epsilon_{it} \sim N(0, \sigma_\epsilon^2)$ . I set the volatility  $\sigma_\xi$  to match the volatility of permanent idiosyncratic income shocks with cyclical skewness:  $\sigma_\xi = 0.122$ . This number falls in the range of values typically used in the life-cycle literature (Gourinchas and Parker, 2002; Cocco et al., 2005). The other parameter values of the income process follow the baseline calibration.

**Alternative government system: proportional transfers and taxes.** I also consider a benchmark case of the model where transfers and taxes are proportional. Many life-cycle models have a replacement income in retirement that is a constant fraction of permanent income just before retirement. If in addition income is taxed at a constant rate, the present value of net labor income is proportional to permanent income  $P$ . When  $\gamma_1 = 0$ , this means that the value function is homothetic in  $P$  and the problem has one less state variable. In that case, the restriction  $b_1 + b_2 = 0$  holds, up to a log-linear approximation. As a benchmark, I therefore consider an alternative specification with proportional government policies: (1) no tax progressivity, i.e.  $\rho = 0$ , and (2) Social Security income is proportional to permanent income in final period, with the same average replacement rate.

## 7 Estimation and Model Fit

### 7.1 Parameter Identification

To examine the quantitative implications of the model, I simulate a sample of 500 000 households that all receive different aggregate and idiosyncratic shocks. Hence, there are no time series and cohort effects in simulated data. I structurally estimate the key parameters of the model. Here, I describe the procedure for estimating these parameters.

**SCF profiles.** I estimate life-cycle profiles of savings and equity shares in the SCF based on the available data from ten waves between 1989 and 2016. I restrict the sample to households with age (of the head) between 25 and 65, with positive income and wealth, and that are not business owners. Sampling weights are adjusted so that each year gets equal weight. As a measure of wealth, I use net worth (*networth*). The equity share is the ratio of equity holdings to financial wealth (*equity/fin*). I normalize wealth by average household income by year, which is \$74K in 2016. To match the earlier empirical analysis, I calculate the age profiles of wealth and equity shares conditional on owning any quasi-liquid retirement wealth (*retqliq* > 0). Since these investors have easy access to stock market investments, I abstract from frictions that limit the participation in financial markets.

Following Ameriks and Zeldes (2004), I construct three-year age groups. Since the SCF is a triennial survey, this means that each cohort moves to the next age group in the following survey

year. It is well known that without additional restrictions, it is impossible to separately identify age, cohort, and time fixed effects (see e.g. Deaton, 1985; Ameriks and Zeldes, 2004). The reason is perfect collinearity: time = cohort + age. As in Fagereng, Guiso, and Pistaferri (2018), I solve this collinearity problem by imposing parametric restrictions on time fixed effects, as proposed by Deaton and Paxson (1994). In particular, I assume that time effects add up to zero and are orthogonal to a linear trend. With this restriction, age effects can be estimated in a regression with controls for cohort fixed effects and restricted time fixed effects.

I run a quantile regression to estimate the profile of median normalized wealth over the life cycle. As target moments for the model, I include median normalized wealth for the age groups with midpoints  $\{28, 34, 40, 46, 52, 58, 64\}$ . In addition, the estimated average equity share at age 50 (age group with midpoint 49) is a model target. The standard errors of these moments are obtained through bootstrapping the procedure. I target the equity share for 50-year old investors to pin down average risk aversion, but I do not target the life-cycle profile of equity shares that is estimated from a cross-sectional comparison of investors. Instead, the model targets individual changes in portfolio allocations in response to wealth shocks. I later use the estimated age profile of equity shares in the SCF as a test of the implications of the model.

**Portfolio regressions.** To quantify variation in risk preferences and the degree of inertia in portfolio rebalancing, the key target moments for the model are the within-person regression estimates of portfolio changes from the portfolio adjustment model in Section 5. To find the model-implied regression coefficients  $b_0$ ,  $b_1$ , and  $b_2$ , I run the same regression in model-simulated data. I select investors with age 30 to 58 and control for a third-order polynomial in initial log financial assets, initial log income (instrumented), a third-order polynomial in the initial log equity share, and a second-order polynomial in age. Appendix Figure A.11 plots the adjustment model regression coefficients estimated from model-simulated data for different values of  $\gamma_1$  and with fixed values for  $\gamma_0$ ,  $\chi$ , and  $\beta$ . The graphs suggest that a positive  $\gamma_1$  and cyclical skewness in labor income are important ingredients to fit the empirical regression coefficients. These implications are later confirmed in formal estimations of the model.

**Permanent income and saving rates.** A well-known stylized feature of the data is that rich households with high lifetime income have higher saving rates than income-poor households (Mayer, 1972; Dynan, Skinner, and Zeldes, 2004). Straub (2019) proposes a method to measure the cross-sectional curvature of consumption in permanent income, and finds estimates of this curvature that imply a significant deviation from the linear relation that is implied by many macroeconomic models. Straub (2019) estimates the following relation:<sup>29</sup>

$$c_{it} = \phi p_{i1} + \delta' X_{it} + \eta_{it}, \quad (41)$$

<sup>29</sup>I focus on the case where the persistent component of individual income is a random walk. Straub (2019) also considers the case with a fixed permanent component  $w_i$  and a mean-reverting process for  $p_{it}$ .

where  $c_{it}$  is log consumption and  $p_{i1}$  is log permanent income upon entering the labor market. Since permanent income is not directly observable, consistent estimates of  $\phi$  can be obtained through running an IV regression of  $c_{it}$  on  $y_{it}$  and  $X_{it}$ , with  $y_{i1}$  as instrument for  $y_{it}$ . Straub (2019) estimates  $\phi$  in PSID data between 1999 and 2013 for households with age 30–65.<sup>30</sup> The estimated value of  $\phi$  is 0.732 with a standard error of 0.05.

Absent heterogeneity in the rate of time preference, saving rates in the life-cycle model are nearly constant across the population.<sup>31</sup> I introduce a simple way to account for variation in saving behavior by wealth in the population by having heterogeneous time discount factors  $\beta_i$  that are correlated with initial permanent income. To find the slope  $\beta_1$  of discount factors with respect to permanent income, I estimate the relation (41) in model-simulated data. The controls include a third-order polynomial in age.

**Objective function.** I estimate the model parameters through indirect inference. In total, there are 12 empirical target moments. The objective is to minimize the weighted distance between moments in the model and in the data:

$$\hat{\alpha} = \arg \min(m(\alpha) - \mu)'W(m(\alpha) - \mu), \quad (42)$$

where  $\alpha$  is the vector of parameters to be estimated,  $m(\alpha)$  are model moments, and  $\mu$  is the vector of moment values in the data. As weighting matrix  $W$ , I use the inverse of the diagonal of the empirical variance-covariance matrix of the moments.

The mapping from parameters to moments is relatively straightforward. The baseline risk aversion parameter  $\gamma_0$  pins down the average equity share, the degree of non-homotheticity in risk aversion  $\gamma_1$  drives the regression coefficients  $b_1$  and  $b_2$ , the probability of updating  $\chi$  is tightly linked to the coefficient  $b_0$  of overall equity share changes on passive equity share changes in the rebalancing regression, the baseline rate of time preferences  $\beta_0$  drives wealth profiles over the life cycle, and heterogeneity in discount rates  $\beta_1$  captures concavity in the cross-sectional relation between consumption and permanent income.

As described below, I will also consider versions of the estimation where some of the moments are excluded from the target vector.

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<sup>30</sup>Straub (2019) focuses on after-tax income. I follow his approach by running the IV regression in model-simulated data with post-tax income.

<sup>31</sup>Note that the model has Epstein-Zin preferences that separate the elasticity of intertemporal substitution from risk aversion. The EIS is assumed to be constant and homogeneous. Furthermore, the bequest motive is homothetic. As a consequence, saving rates are nearly unaffected by the level of wealth. In fact, when  $\gamma_1 = 0$  (CRRA preferences) and transfers and taxes are proportional, the value function is homothetic in permanent income and only the relative proportion of financial wealth to human capital matters for consumption and saving rates. This neutrality is broken by the Social Security system where replacement rates decrease in income, by progressive income taxation, and by DRRA preferences that imply that poorer households are more risk averse and therefore have a greater demand for precautionary savings. However, these deviations from a neutral model are not sufficient to generate the empirical degree of concavity  $\phi$ .

## 7.2 Estimation Results

**Parameter estimates.** Table 8 reports the estimated parameters for different specifications of the model. Column (1) reports the estimated parameters for the baseline model that has countercyclical tail risk income, progressive taxes and a realistic Social Security system with non-proportional replacement rates. The baseline coefficient of relative risk aversion (approximately equal to average risk aversion in model simulations) is 5.7, which falls within typical ranges and is lower than the high risk aversion coefficient that some portfolio choice models need to match equity shares (see e.g. [Fagereng et al., 2018](#)). Most importantly, risk preferences are estimated to be strongly non-homothetic:  $\gamma_1 = 0.31$ . Further, the annual probability of portfolio updating is estimated at 15%, the baseline discount rate is slightly below 0.95, and discount rates are heterogeneous ( $\beta_1 > 0$ ) to match heterogeneity in saving rates. Investors are somewhat more passive in the model than what is observed in the data; the other moments of the fitted model fall within the confidence bounds of the empirical moments.

Columns (2)–(4) report parameter estimates for alternative specifications of the model with progressive taxes and non-proportional retirement replacement income. Column (2) restricts preferences to be of the CRRA form and excludes the portfolio responses to wealth shocks from the target moments. The resulting model fits the cross-sectional moments but is inconsistent with the within-person portfolio evidence. Column (3) considers an alternative model specification where income is not subject to tail risk. While the implications for risk preferences are similar, this version fits the data less well. Safer labor income raises the desire to tilt financial wealth to risky assets (hence, higher risk aversion) and weakens the precautionary saving motive (hence, less time discounting). Column (4) imposes homogeneity in time discount rates  $\beta$ . The implied non-homotheticity in risk preferences is even stronger, but consistent with [Straub \(2019\)](#), the model does not fit the degree  $\phi$  of cross-sectional concavity in consumption by income levels.

The baseline model deviates from a homothetic model not just by allowing for DRRA preferences, but also by having non-proportional government policies. As a result of progressive taxes and retirement replacement rates that decline in income levels, increases in labor income translate less than one-for-one to increases in human capital. I find that the qualitative results of the model do not depend on this particular form of government policies. Columns (5)–(7) report estimated parameters for a more traditional setup of the model where transfers and taxes are proportional to permanent income, and without heterogeneity in discount rates. Column (5) shows that this version of the model closely matches all targets except for heterogeneity in saving rates, and still implies a significant degree of non-homotheticity in risk preferences. Column (6) shows that a restriction version with  $\gamma_1 = 0$  (CRRA preferences) again does not fit the portfolio evidence. In this fully homothetic version of the model, for a given value of cash on hand relative to permanent income, there are no effects of permanent income  $P$  on consumption and saving rates. Indeed, we find that  $\phi = 1$  and that the portfolio regression coefficients add up to a number that is close to zero. Finally, column (7) shows that also in the model with proportional government policies, a standard specification of labor income without tail risk provides a worse

Table 8: Parameter Estimates

	Non-proportional govt policies				Proportional govt policies			Data
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	
Baseline risk aversion $\gamma_0$	5.7118 (0.0786)	6.5857 (0.0823)	6.7061 (0.1144)	5.8311 (0.0854)	5.1999 (0.0617)	5.6315 (0.0773)	6.0066 (0.0982)	
Non-homotheticity in risk aversion $\gamma_1$	0.3106 (0.0158)	0	0.2697 (0.0152)	0.3999 (0.0167)	0.2245 (0.0168)	0	0.1567 (0.0162)	
Calvo probability $\chi$	0.1520 (0.0039)	0.1661 (0.0058)	0.1255 (0.0034)	0.1698 (0.0043)	0.1825 (0.0055)	0.1608 (0.0056)	0.1469 (0.0039)	
Baseline discount rate $\beta_0$	0.9482 (0.0017)	0.9566 (0.0016)	0.9610 (0.0014)	0.9423 (0.0020)	0.9445 (0.0020)	0.9507 (0.0019)	0.9551 (0.0018)	
Slope of discount rate $\beta_1$	4.2358 (0.3632)	3.1067 (0.4640)	3.6189 (0.3905)	0	0	0	0	
Income process	Tail risk	Tail risk	No tail risk	Tail risk	Tail risk	Tail risk	No tail risk	
Targets								
Age profile of financial wealth	Y	Y	Y	Y	Y	Y	Y	
Equity share at age 50	Y	Y	Y	Y	Y	Y	Y	
Portfolio regression $b_0$	Y	Y	Y	Y	Y	Y	Y	
Portfolio regression $b_1, b_2$	Y	Y	Y	Y	Y		Y	
Saving regression $\phi$	Y	Y	Y					
Value of objective (full)	50.880	375.064	98.611	87.685	57.928	434.184	97.002	
Value of objective (limited)	50.880	16.038	98.611	64.050	25.620	30.465	64.933	
Equity share at age 50	0.4170	0.4259	0.4213	0.4409	0.4432	0.4238	0.4143	0.4309 (0.0111)
Portfolio regression $b_0$	0.8646	0.8393	0.8791	0.8537	0.8337	0.8390	0.8551	0.8372 (0.0059)
Portfolio regression $b_1$	0.0568	0.0196	0.0580	0.0534	0.0487	0.0134	0.0541	0.0525 (0.0021)
Portfolio regression $b_2$	-0.0138	-0.0329	-0.0186	-0.0130	-0.0109	-0.0187	-0.0179	-0.0095 (0.0023)
Saving regression $\phi$	0.6480	0.7338	0.6881	0.9731	1.0142	1.0001	1.0132	0.7300 (0.0500)

Notes: This table reports the results of structural estimations of different versions of the model. I first report the estimated parameter values and their corresponding standard errors, I then indicate which moments were targeted in the estimation, and finally I report the value of the objective and the key moments of the model evaluated at the estimated parameter values.

fit to the data.

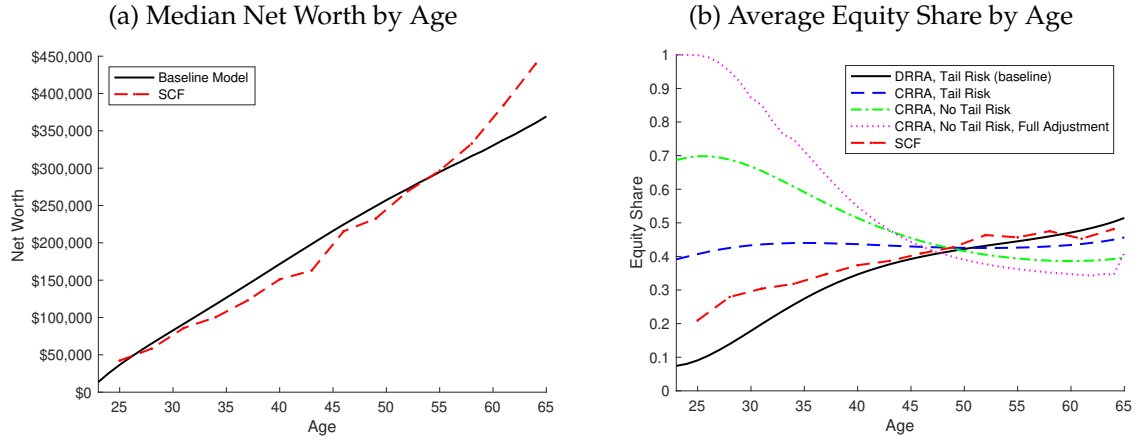
The estimation results are robust to using a different weighting matrix for the moment conditions or changing fixed parameters in the model. In Appendix A.4.3, I illustrate and discuss the sensitivity of the estimated parameters to the empirical moments, using the method proposed by Andrews, Gentzkow, and Shapiro (2017).

**Model fit.** The results in Table 8 highlight that the model with DRRA preferences fits the reduced-form portfolio regression estimates, while the model with CRRA preferences does not. Although the qualitative implications of the model do not depend on the specific income process, the model with tail risk in labor income fits the data significantly better.

Next, I look at the implications of the estimated preferences for life-cycle profiles in the model. First, Figure 9a plots median asset holdings over the life cycle in the model and in the SCF (conditional on holding retirement wealth). The age profile of savings is targeted in model



Figure 9: Life-Cycle Profiles of Savings and Equity Shares



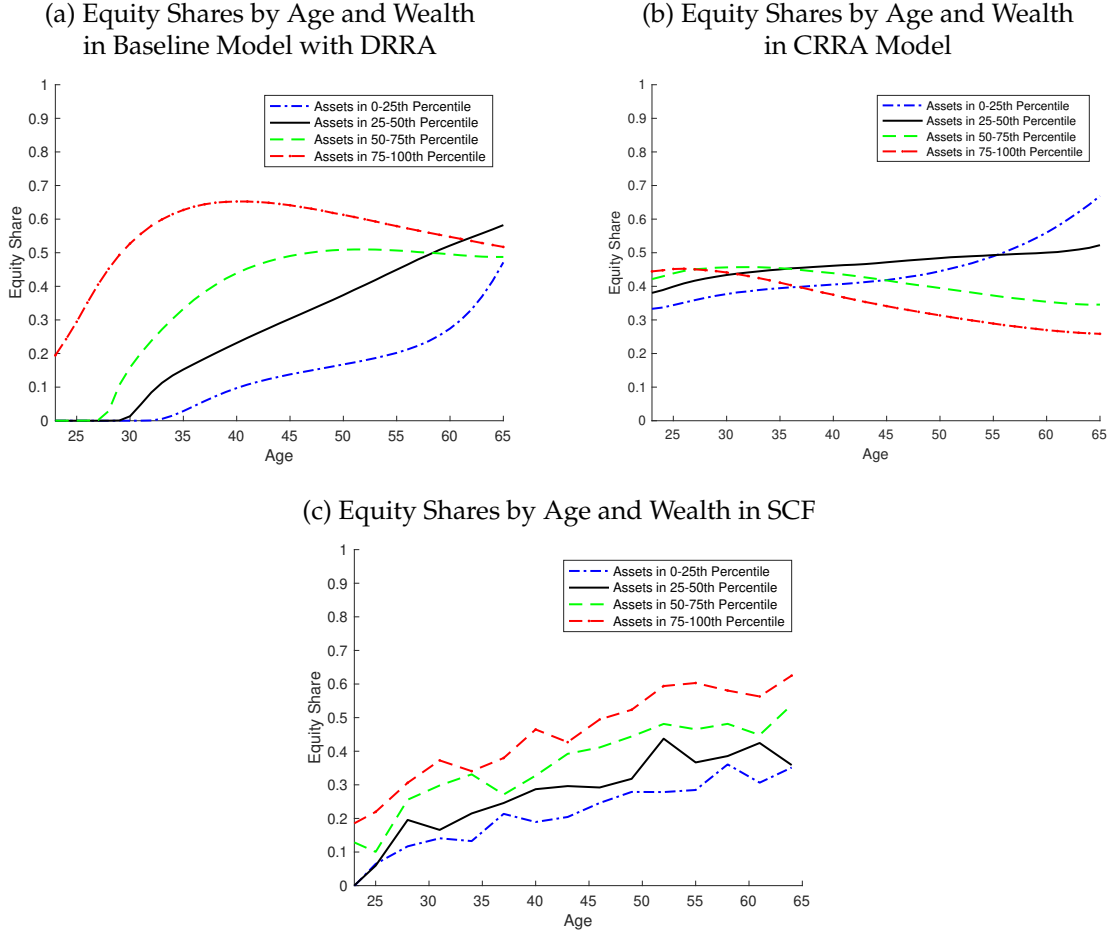
Notes: Panel (a) plots median net worth by age in the baseline model and in the SCF (RIs). Panel (b) compares the age profiles of equity shares in various specifications of the model to the SCF.

estimations, and the fitted model matches the data well.

Second, the profile of equity shares over the life cycle has been studied extensively in the household finance literature. While empirically the age profile of equity shares tends to be relatively flat or even upward sloping, in traditional life-cycle models the equity share sharply declines with age. One possible explanation for the age profile of equity shares is DRRA preferences (Wachter and Yogo, 2010), but there are many alternatives, such as cointegration between labor income and dividends (Benzoni et al., 2007) and disaster risk in stock markets (Fagereng et al., 2018). Distinguishing these theories of portfolio choice is difficult with cross-sectional data. I provide direct empirical evidence in support of DRRA preferences using within-person portfolio changes.

Even though I only target the level of the equity share at age 50 and not the age profile, Figure 9b shows that the baseline model with DRRA generates an age profile of equity shares that is upward sloping and closely aligns with the age profile of equity shares of households in the SCF that are saving for retirement. Figure 9b highlights three main channels in the model through which the life-cycle profile of equity shares deviates from the traditional downward-sloping curve. With CRRA preferences, no tail risk in labor income, and full portfolio adjustment each year, the age profile of equity shares is strongly downward sloping, starting at 100% equity for young households. With infrequent, time-dependent portfolio adjustment, the equity share profile is still downward sloping but flatter. Accounting for tail risk in labor income, the fitted model generates a flat age profile of equity shares. Finally, equity shares increase with age and are closest to the data when income tail risk is combined with non-homothetic preferences in the baseline version of the model.

Figure 10: Equity Shares over the Wealth Distribution



Notes: These figures plots average equity shares by age and wealth groups. Panel (a) shows the results for the baseline model with DRRA preferences, panel (b) shows the outcomes from the restricted model with CRRA preferences, and panel (c) plots the empirical equivalents from the SCF.

### 7.3 DRRA and Heterogeneity in Expected Returns

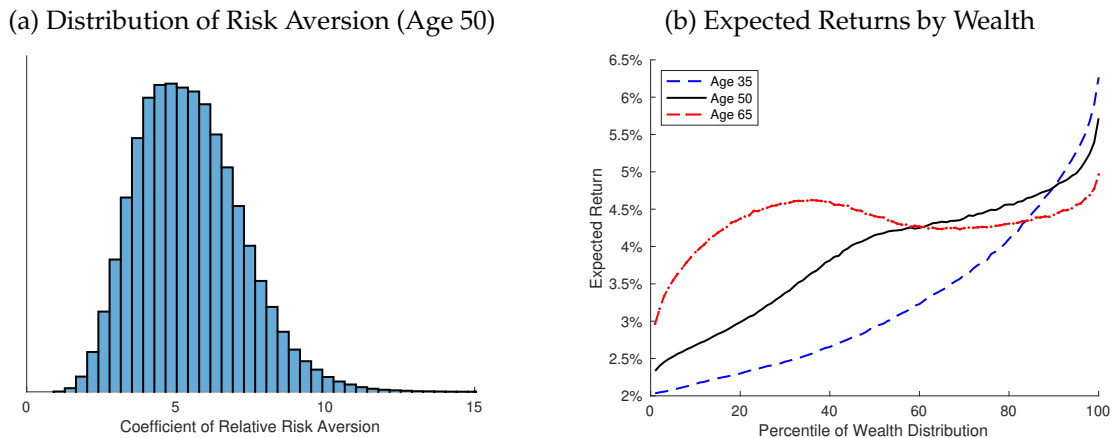
The parameter  $\gamma_1$  drives the curvature of the certainty equivalent aggregation function  $J$  in the Epstein-Zin framework. A positive  $\gamma_1$  implies that preferences are DRRA. What is the implied elasticity of risk aversion with respect to wealth? I calculate the relative risk aversion of households from the local curvature evaluated at current value function  $V_{it}$ :  $RRA_{it} = \gamma_0 (V_{it}/\kappa\beta_i)^{-\gamma_1}$ . I estimate the following specification in model-simulated data:

$$\Delta \log RRA_{it} = \zeta \cdot \Delta p_{it} + \delta' X_{i,t-1} + \eta_{it}. \quad (43)$$

The controls  $X_{i,t-1}$  include a polynomial in age. Estimations of the model parameters yield a significant degree of non-homotheticity in risk preferences: the average elasticity  $\zeta$  of risk aversion with respect to permanent income is  $-0.17$ .

Since risk aversion decreases in wealth, wealthier households desire a bigger portfolio share

Figure 11: Heterogeneity in Risk Aversion and Returns to Wealth



Notes: This figure illustrates heterogeneity in risk aversion and expected returns by wealth in the baseline model. Panel (a) plots the distribution of the coefficient of relative risk aversion at age 50. Panel (b) plots expected returns over the wealth distribution at age 35, age 50, and age 65.

of equity, *ceteris paribus*. Figure 10a plots the average equity share by age and wealth in the baseline model with DRRA, where households of each age are split in four groups based on their rank in the asset distribution. Except close to retirement, equity shares monotonically increase with wealth. Having DRRA preferences is crucial for generating this pattern: Figure 10b shows a weak and non-monotonic relation between wealth and risk-taking behavior in the restricted model with CRRA preferences. Figure 10c plots equity shares by age and wealth in the SCF. The data show a pattern that is consistent with the implications of the DRRA model – households with high net worth hold considerably higher equity shares than households with lower net worth (see also Wachter and Yogo, 2010, who include housing equity in the risky portfolio and find a bigger dispersion in risk taking over the wealth distribution).

To further illustrate the quantitative significance of DRRA risk preferences, estimated from within-person changes, Figure 11a plots the resulting cross-sectional distribution of relative risk aversion in the baseline model specification for agents of age 50. Wealth dispersion leads to a sizable variation in risk aversion across investors, with a reasonable range: the largest mass is around a risk aversion of 5, the 5th percentile is 2.9, and the 95th percentile is 8.5.

Figure 11b plots expected returns by wealth for three different ages. The positive relation between wealth and average portfolio returns matches cross-sectional patterns in the data (Fagereng et al., forthcoming). The range of expected returns by net worth, conditional on age, nearly spans the full equity premium: households in the lowest percentile of the net worth distribution invest the majority of their financial wealth in the risk-free asset, while households in the top of the net worth distribution invest only in equity. Since all agents in the model have access to two assets, a risk-free asset and a stock market index, these patterns are fully driven by differences in risk-taking behavior as opposed to differences in technologies.

## 8 Aggregate Implications

I exploit the estimated life-cycle consumption and portfolio choice model to analyze aggregate implications of DRRA preferences. First, I examine the implications of heterogeneity in expected returns for wealth inequality. Second, I look at the effect of rising income inequality on wealth inequality and asset prices.

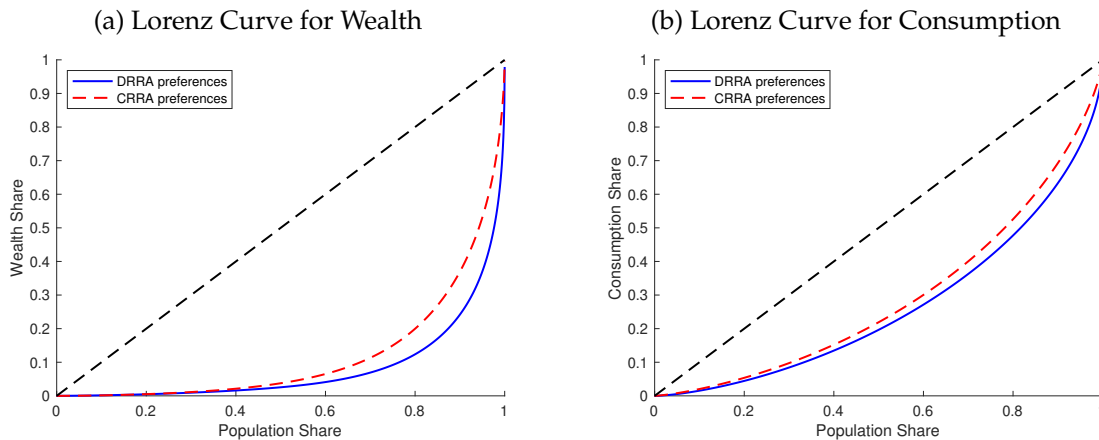
### 8.1 Wealth Inequality

A large body of research exists on factors that drive the degree of inequality in the wealth distribution that is observed in the data. The literature points to three main drivers that are important for understanding the level and the dynamics of wealth inequality. First, motivated by the Aiyagari-Beweley framework, many papers have focused on the role played by idiosyncratic and uninsurable labor income to generate tails in the wealth distribution (e.g. [Castaneda, Diaz-Gimenez, and Rios-Rull, 2003](#)). Properties of the labor income distribution typically carry over to similar properties for the wealth distribution. Empirically, however, income inequality is an order of magnitude lower than wealth inequality. Therefore, another branch of the literature has focused on differential saving rates as a second factor, and has combined earnings heterogeneity with heterogeneity in saving rates across wealth levels to generate a thick tail in the wealth distribution (e.g. [De Nardi, 2004](#); [Carroll, 2000](#)). A third factor that has been shown to be instrumental in generating a skewed wealth distribution with sufficient fluctuations in tail inequality is heterogeneity in returns to wealth, especially if those returns are positively correlated with wealth itself ([Benhabib, Bisin, and Zhu, 2011](#); [Gabaix, Lasry, Lions, and Moll, 2016](#); [Benhabib, Bisin, and Luo, 2019](#)). [Fagereng et al. \(forthcoming\)](#) provide empirical evidence on heterogeneity in expected returns across the wealth distribution. An open question is what the source of heterogeneity in expected returns is. Heterogeneity in returns to wealth could be due to differences in risk preferences, or due to technological features of asset returns such as fixed costs of high-return investment opportunities or economies of scale in wealth management. Understanding these differences is important for designing policies that target wealth accumulation.

The life-cycle model estimated in the previous section does not assume any differences in investment technologies across agents. However, agents vary in the amount of compensated risk they take due to DRRA preferences. Non-homotheticity in risk preferences implies that there is a two-way relation between wealth and equity demand. Since risk aversion decreases in wealth, richer households invest a larger share of their portfolios in equity. Because of the equity premium of 4.5%, wealth inequality gets further amplified through differences in average portfolio returns. As displayed in the previous section, average returns indeed vary widely over the wealth distribution in the model.

I find that DRRA preferences have significant implications for the cross-sectional distribution of wealth. By targeting the within-person portfolio responses to wealth shocks and the cross-sectional relation between consumption and permanent income, the model generates an

Figure 12: Lorenz Curves



Notes: This figure shows Lorenz curves for wealth and consumption in the baseline model. The Lorenz curve illustrates the share of wealth that the bottom  $x\%$  of the distribution hold, as a function of  $x$ . The further away the curve is from the 45-degree line, the more unequal the distribution is.

(untargeted) wealth distribution with large inequality: Table 9 reports that the wealth share of the top 1% is 37.0%. An important contributor to this large wealth inequality is that equity holdings are concentrated in the hands of the rich. Figure 12 compares the Lorenz curves for DRRA preferences and CRRA preferences. In an alternative estimation of the model where risk preferences are CRRA, the top 1% wealth share drops to 20.4%. Similarly, in a version of the model where the equity premium is set to zero, the top 1% wealth share is 19.4%.

## 8.2 Effects of Rising Income Inequality

The last few decades have seen an increase in income inequality in the United States (Autor, Katz, and Kearney, 2008). An important force behind increased income inequality is an increase in the dispersion of permanent income levels of new cohorts (Guvenen et al., 2018). With a concave relation between permanent income and consumption, an increase in permanent income inequality leads to increased wealth inequality (Straub, 2019). DRRA preferences amplify wealth inequality by generating heterogeneity in expected returns across the wealth distribution. In this section, I ask two questions: (1) What is the effect of rising income inequality on wealth inequality in the presence of multiple assets combined with non-homothetic risk preferences? (2) What are the long-term effects of rising inequality on asset prices?

I use the estimated life-cycle model to run a counterfactual analysis of the effects of rising inequality on asset demand and the wealth distribution. As input, I take the rise in income inequality in the SCF between 1989 and 2016. Recall that in the baseline model, the dispersion in initial income is calibrated to match the Gini coefficient of income in the SCF in 2016, which is 0.551. I now compare simulations of the model to a version where the dispersion in initial income is calibrated to match the 1989 Gini coefficient of income of 0.453. All other parameters are held constant. Table 9 reports the effects of rising income inequality on the wealth distribution. In the

Table 9: Inequality in the Model

	Gini	Share of wealth or consumption in top				
		Top 1%	Top 5%	Top 10%	Top 25%	Top 50%
<i>Wealth inequality</i>						
Baseline model	0.838	37.0%	63.0%	75.8%	90.8%	97.5%
CRRA preferences	0.763	20.4%	47.0%	63.1%	85.0%	96.3%
No equity premium	0.757	19.4%	46.9%	63.3%	85.0%	95.6%
Homogeneous $\beta$	0.547	12.9%	31.4%	44.1%	65.5%	84.2%
1989 income inequality	0.714	22.0%	45.3%	59.7%	80.4%	93.1%
<i>Consumption inequality</i>						
Baseline model	0.471	10.8%	25.5%	36.7%	58.4%	80.4%
CRRA preferences	0.417	6.2%	19.4%	30.7%	54.1%	78.1%
No equity premium	0.360	4.6%	15.4%	25.6%	48.5%	74.9%
Homogeneous $\beta$	0.464	8.3%	22.7%	34.3%	57.5%	80.7%
1989 income inequality	0.412	8.5%	21.6%	32.1%	53.7%	77.0%

*Notes:* This table reports statistics on inequality in the distribution of wealth and consumption for different versions of the model. Starting point is the baseline estimation of the model with non-proportional transfers and taxes. The first deviation is a restricted parameterization where  $\gamma_1 = 0$ , so that risk preferences are CRRA. In a second deviation, I set the equity premium to zero so that investors allocate all financial assets to the risk-free security. The third deviation is a restricted parameterization where  $\beta_1 = 0$  so that discount rates are homogeneous. Finally, I consider an alternative calibration where the distribution of initial income is chosen so that the Gini coefficient of income in the model matches the equivalent in the 1989 SCF (as opposed to the 2016 SCF).

model, the top 1% wealth share rises from 22.0% to 37.0%.

Holding the risk-free rate and the equity premium constant, the increase in inequality leads to additional demand for the risk-free asset and, in particular, for equity. The demand for equity as a fraction of overall wealth in the economy increases from 58% to 71%. As a second step, I run a similar exercise as in [Catherine \(2019\)](#): I calculate the change in equity premium that offsets this increase in demand for equity. I assume that supply of the two securities adjusts such that the aggregate asset value composition in the economy remains constant. Specifically, in the model with income inequality at 2016 levels, I search for the value of the equity premium that moves the aggregate equity share back to its level in the model with inequality at 1989 levels. The difference in risk premium captures the effect of the rise in income inequality. I find that increased inequality has a significant low-frequency effect on asset prices. The model implies that rising inequality over the past few decades has led to increased demand for equity that is equivalent to a decrease in the equity premium of 0.69 percentage points – a decline by 15%.<sup>32</sup>

<sup>32</sup>The estimated non-homotheticity in risk tolerance also provides qualitative support for asset pricing models based on cross-sectional or time-series variation in risk aversion that fit important asset pricing facts (e.g. [Campbell and Cochrane, 1999](#); [Chan and Kogan, 2002](#); [Gârleanu and Panageas, 2015](#)). However, in Appendix [A.4.4](#) I show that the time-series variation in aggregate risk aversion at business cycle frequencies in the estimated life-cycle model is quantitatively limited.

## 9 Conclusion

A rich theoretical literature on household portfolio choice studies optimal asset allocations under a wide range of assumptions on preferences and financial profiles. Recent life-cycle models extend the assumptions of traditional models by including realistic features of the household problem such as non-diversifiable idiosyncratic income risk, borrowing constraints, and time-varying investment opportunities. Due to the limited availability of panel data that meets the demanding requirements for testing these theories at the micro level, the literature has often relied on cross-sectional patterns in the data, mostly from surveys, to calibrate or estimate key model parameters. This identification strategy requires restrictive assumptions on differences across individuals. In parallel, existing micro-level studies of portfolio choice in panel data have focused on qualitative tests of some of the main channels of theoretical models in reduced-form specifications. The quantitative implications of empirical patterns in household portfolio choice are largely unexplored.

In this paper, I provide new evidence on investor portfolio changes in response to financial changes in a large sample of U.S. retirement investors that (1) provide a qualitative test of whether risk aversion decreases in wealth, and (2) guide a quantitative investigation of portfolio choice behavior. I measure how portfolio risk taking changes in response to fluctuations in labor income and returns to financial wealth. While the effect of financial wealth on risk taking is in itself not informative about risk preferences, the combined effects of income growth and portfolio returns provide a test of CRRA versus DRRA preferences. Controlling for infrequent portfolio adjustment and ex-ante differences across individuals, I find that positive and persistent shocks to income lead to an increase in the equity share of investor portfolios. Increases in financial wealth due to realized returns lead to a small decline in the equity share. The positive net effect of income growth and portfolio returns on equity shares conflicts with the prediction of a standard homothetic life-cycle model and suggests that risk aversion decreases in wealth.

Using these empirical findings, I structurally estimate the parameters of a life-cycle consumption and portfolio choice model that allows for DRRA preferences and accounts for business cycle variation in the tail risk that is embedded in human capital. I find that the model is able to closely match the empirical findings with a significant degree of non-homotheticity in risk preferences – the average elasticity of risk aversion with respect to permanent income is  $-0.17$ . I use the model to study the distributional and aggregate consequences of DRRA preferences. The model has important quantitative implications for inequality. Decreasing risk aversion in wealth concentrates equity in the hands of the wealthy and leads to a cross-sectional relation between wealth and expected returns that is consistent with patterns in the data. The wealth share of the top 1% in the model nearly doubles due to heterogeneity in expected returns to financial wealth. The model further suggests that rising inequality in the U.S. has led to increased demand for equity that is equivalent to a decrease in the equity premium by 15%.



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# Appendix

## A.1 Stylized Model Derivations

**Portfolio choice.** Let  $Z = W + P$  be total wealth at time  $t = 0$ . Note that

$$\begin{aligned} W_1 &= W(R^f + \theta(R^e - R^f)) + P(R^f + \phi(R^e - R^f)) \\ &= (W + P) \left\{ R^f + \left( \theta \frac{W}{W + P} + \phi \frac{P}{W + P} \right) (R^e - R^f) \right\} \\ &= (W + P)(R^f + \alpha(R^e - R^f)) \equiv ZR^{tot}. \end{aligned} \quad (\text{A.1.1})$$

Hence,  $\theta = \alpha + (\alpha - \phi) \frac{P}{W}$ .

Let  $V_1(W_1) = \frac{(W_1 - X_1)^{1-\gamma}}{1-\gamma}$  be the value function as a function of wealth at time 1. The standard first-order condition of the portfolio choice problem is

$$\mathbb{E}[V_1'(W_1)R^e] = \mathbb{E}[V_1'(W_1)R^f]. \quad (\text{A.1.2})$$

Applying the approximation  $\log \mathbb{E}[e^y] \approx \mathbb{E}[y] + \frac{1}{2}\text{Var}[y]$ , which holds with equality when  $y$  is normally distributed, to the left hand side and right hand side of (A.1.2) yields

$$\mu_e - r_f + \frac{1}{2}\sigma_e^2 \approx -\text{Cov}[\log R^e, \log V_1']. \quad (\text{A.1.3})$$

Let  $X = X_1/R^f$  be the present value of the subsistence level, and let smaller case letters denote logs, with  $\tilde{w}_1 = \log(W_1 - X_1)$ . A log-linearization around  $R^e = R^f$  gives

$$\tilde{w}_1 = k' + \frac{Z}{Z - X} r_{tot}, \quad (\text{A.1.4})$$

where  $k'$  is a log-linearization constant. Plugging this into the approximated Euler equation gives

$$\mu_e - r_f + \frac{1}{2}\sigma_e^2 \approx \alpha \frac{Z}{Z - X} \gamma \sigma_e^2. \quad (\text{A.1.5})$$

The solution to the portfolio choice problem is

$$\alpha = \frac{\mu_e - r_f + \frac{1}{2}\sigma_e^2}{\gamma \sigma_e^2} \cdot \frac{Z - X}{Z} = \bar{\alpha} \left( 1 - \frac{X}{W + P} \right), \quad (\text{A.1.6})$$

where

$$\bar{\alpha} = \frac{\mu_e - r_f + \frac{1}{2}\sigma_e^2}{\gamma \sigma_e^2}. \quad (\text{A.1.7})$$



As a fraction of financial wealth, the optimal portfolio share  $\theta$  is given by

$$\begin{aligned}\theta &= \alpha + (\alpha - \phi) \frac{P}{W} \\ &= \bar{\alpha} + (\bar{\alpha} - \phi) \frac{P}{W} - \bar{\alpha} \left(1 + \frac{P}{W}\right) \frac{X}{W + P}.\end{aligned}\tag{A.1.8}$$

**Comparative statics.** Further log-linearization of (A.1.8) around  $R^{pf} = R^f$  and  $R^p = R^f$  yields

$$\begin{aligned}\log \theta \approx & k + \lambda_1 \frac{\bar{W}}{\bar{W} + \bar{P}} (\log P - \log R^{pf}) \\ & - \lambda_2 \left( \frac{\bar{P}}{\bar{W} + 2\bar{P}} \frac{\bar{W}}{\bar{W} + \bar{P}} (\log P - \log R^{pf}) - \frac{\bar{W}}{\bar{W} + 2\bar{P}} \log R^{pf} - \frac{2\bar{P}}{\bar{W} + 2\bar{P}} \log P \right),\end{aligned}\tag{A.1.9}$$

where  $k$  is a log-linearization constant, and

$$\begin{aligned}\bar{\theta} &= \bar{\alpha} - \bar{\alpha} \left(1 + \frac{\bar{P}}{\bar{W} + \bar{P}}\right) \frac{\bar{X}}{\bar{W} + 2\bar{P}} + (\bar{\alpha} - \phi) \frac{\bar{P}}{\bar{W} + \bar{P}}, \\ \lambda_1 &= \frac{\bar{\alpha} - \phi}{\bar{\theta}} \cdot \frac{\bar{P}}{\bar{W} + \bar{P}}, \quad \lambda_2 = \frac{\bar{\alpha}}{\bar{\theta}} \left(1 + \frac{\bar{P}}{\bar{W} + \bar{P}}\right) \frac{\bar{X}}{\bar{W} + 2\bar{P}}.\end{aligned}\tag{A.1.10}$$

After rearranging, we arrive at

$$\log \theta \approx k + (\kappa_1 \lambda_1 + (1 - \kappa_2) \lambda_2) \log P + (-\kappa_1 \lambda_1 + \kappa_2 \lambda_2) \log R^{pf},\tag{A.1.11}$$

where

$$\kappa_1 = \frac{\bar{W}}{\bar{W} + \bar{P}}, \quad \kappa_2 = \frac{\bar{W}}{\bar{W} + 2\bar{P}} \left(1 + \frac{\bar{P}}{\bar{W} + \bar{P}}\right).\tag{A.1.12}$$

**Extension to dynamic model.** The state variables in a standard homothetic life-cycle model are age  $t$  and relative cash on hand  $w_{it}$ . The optimal portfolio is  $\theta_{it} = \Theta(w_{it}, t)$ . The dynamics of  $w_{it}$  are given by (10). Consider a log-linearization of  $\theta_{i,t+1}$  around  $R_{i,t+1}^{pf} = R^f$ ,  $P_{i,t+1}/P_{it} = R^f$ , and  $\epsilon_{i,t+1} = 0$ . Let

$$\bar{w}_{i,t+1} = (w_{it} - c_{it}) \frac{G_t}{G_{t+1}} + 1 - \tau.\tag{A.1.13}$$

We obtain the approximation

$$\log \theta_{i,t+1} = k_{it} + \frac{\Theta_w(\bar{w}_{i,t+1}, t+1) \bar{w}_{i,t+1}}{\Theta(\bar{w}_{i,t+1}, t+1)} (\rho_{it} \log R_{i,t+1}^{pf} - \rho_{it} \Delta \log P_{i,t+1} + (1 - \rho_{it}) \epsilon_{i,t+1}),\tag{A.1.14}$$

where  $k_{it}$  is a log-linearization constant that depends on time- $t$  information, and

$$\rho_{it} = \frac{(w_{it} - c_{it}) \frac{G_t}{G_{t+1}}}{\bar{w}_{i,t+1}}.\tag{A.1.15}$$

Hence, we get

$$\log \theta_{i,t+1} = k_{it} + b_{1,i,t} \Delta \log P_{i,t+1} + \underbrace{(-b_{1,i,t})}_{b_{2,i,t}} \log R_{i,t+1}^{pf} + b_{3,i,t} \epsilon_{i,t+1}, \quad (\text{A.1.16})$$

where

$$\begin{aligned} b_{1,i,t} &= -\frac{\Theta_w(\bar{w}_{i,t+1}, t+1) \bar{w}_{i,t+1}}{\Theta(\bar{w}_{i,1}, t+1)} \rho_{it} \\ b_{3,i,t} &= \frac{\Theta_w(\bar{w}_{i,t+1}, t+1) \bar{w}_{i,t+1}}{\Theta(\bar{w}_{i,1}, t+1)} (1 - \rho_{it}). \end{aligned} \quad (\text{A.1.17})$$

## A.2 Data Construction

This section includes additional details on the construction of the dataset.

**Asset classes.** Investor portfolios are composed of positions in funds, individual securities, and annuities. Holdings are assigned to four different asset classes based on product descriptions: equity, fixed income, cash and cash-like securities, and alternative assets.<sup>33</sup> Equity holdings consist of pure equity funds, directly held equity, and the equity portion of multi-asset class funds. The fixed income category includes bond funds, individual bonds, and the portion of multi-asset class funds that is not allocated to equity. The category of cash and cash-like securities includes money market funds and liquid short-term debt.

Mixed-assets funds, such as target date funds, are split into an equity component and a fixed income component based on fund equity shares. I use quarterly data on fund asset compositions from the CRSP Survivor-Bias Free US Mutual Fund database if available, and complement this with internally available quarterly target equity shares on other mixed-asset funds.

**International exposure.** To characterize international equity exposures in investor portfolios, equity holdings are divided into a domestic and an international component. Pure equity funds are characterized as either domestic or international based on internal product descriptions. The equity portion of mixed-asset funds is treated as a domestic equity investment. For individual securities, I set the location to international if it is a foreign security (i.e., has a foreign ISIN) or if the company is incorporated outside of the US according to Compustat, and to domestic otherwise. The international share of equity is defined as the ratio of international equity to total portfolio equity holdings.

**Returns.** I compute realized returns using two methods. The first method is based on external return data. Observed portfolio holdings are linked to external data on realized returns from CRSP stock, treasury, and mutual fund return files, as well as WRDS corporate bond returns,

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<sup>33</sup>Investment products with insufficient detail to categorize holdings are excluded. Average holdings in these assets are less than 1.5% of total (investable) assets.

using CUSIP identifiers that are available for all public securities and funds in the data. Assets in the cash and cash-like securities class are treated as risk-free assets and are assigned the risk-free rate (one-month Treasury bill rate) as return. While this method provides a return for the large majority of assets in the data, the returns on some assets are not available (e.g. non-public funds).

In a second method, I compute yearly portfolio returns from annual portfolio holdings and transactions at the security level. In particular, I calculate the price appreciation from positions with constant holdings in the asset. To this price appreciation I add the dividends that were paid out over the year, assuming no reinvestment. Using this method, I get a nearly complete coverage of asset returns. The results that are reported in the paper are based on this second method. The results are robust to using the publicly available returns from the first method.

**Market betas.** CAPM market betas are estimated from monthly regressions of excess asset returns on excess market returns. A market beta is assigned to funds and securities that have at least 24 monthly return observations.

## A.3 Additional Empirical Analysis

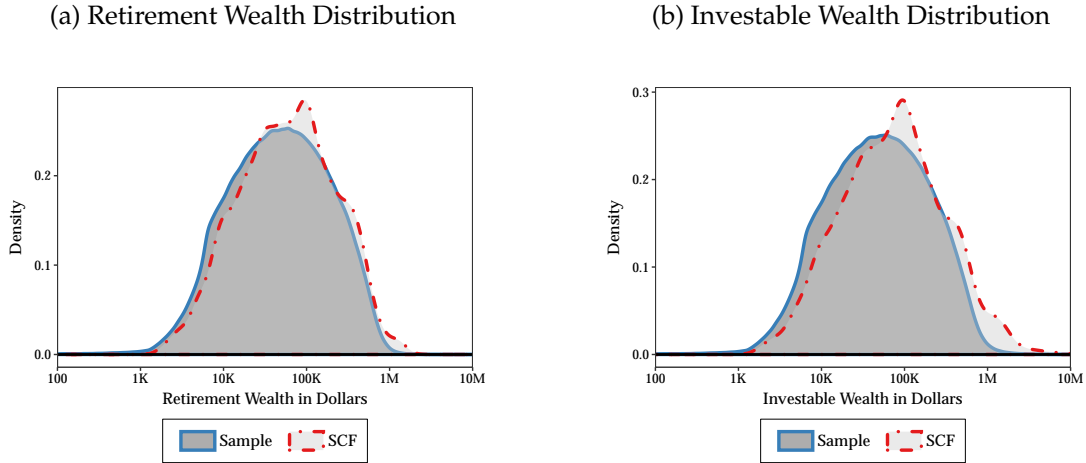
### A.3.1 Wealth Composition of RIs

While the data on investor portfolios are rich on information about asset holdings and characteristics of a large sample of typical U.S. retail investors, a limitation of the data is that only wealth that investors hold with this particular firm is observable. In this section, I use the SCF to summarize the main sources of wealth that are missing and the potential implications for the analysis.

The first type of missing wealth is wealth that falls under the categories that are observable in the data but that is outside of the sample: investable wealth held with other firms or by other members of the household. Figure A.1 compares the distribution of total investable wealth at the household level in the SCF to individual investor wealth in the sample. The distribution of total investable wealth in the sample lines up reasonably well with the distribution across households in the SCF, although the sample misses some non-retirement wealth of wealthy households. The first row of Table A.1 confirms that for most retirement investors, retirement wealth is the main form of investable wealth. The mean (median) share of retirement wealth in investable wealth is 88% (100%). I conclude that the data provide an accurate representation of how investable wealth is allocated for a large sample of U.S. investors.

The second type of missing wealth is other sources of wealth besides investable wealth. Table A.1 presents statistics on the composition of total net worth for retirement investors in the SCF of age 30–58 that earn a wage income above the minimum wage income and have a positive net worth of at least one thousand dollars. The mean (median) share of retirement wealth in total net worth is 51% (40%). The main source of missing wealth is housing wealth. Home equity accounts on average for 37% of wealth. The second source of missing wealth is bank accounts

Figure A.1: Wealth in Firm Data (Individual) and SCF (Household)



Notes: This figure plots the distribution of individual wealth in the sample of RI investors versus the distribution of household wealth for RI investors in the SCF. Panel (a) displays the distribution of retirement wealth and panel (b) displays the distribution of total investable wealth.

and other liquid savings, accounting on average for 10% of net worth. Non-retirement investable wealth covers 7% of net worth. These types of wealth that are missing in the data likely have different risk characteristics than investable wealth. In particular, we would typically expect equity exposures to be higher in investable wealth than in missing sources of wealth. Therefore, in the empirical analysis I allow the measured portfolio equity share in the data to differ from the equity share of the full portfolio by a scaling factor – uncorrelated with wealth changes after controlling for observables – by running the main regressions in logs. In the model estimation, I include the average equity share of total financial wealth at age 50 from the SCF as a target.

The main question raised by these summary statistics is how the analysis changes in the presence of housing equity. First, if households would permanently live in the same house and consume its services, there would be no effect of housing on the allocation of investable wealth. Second, when accounting for housing as an investment vehicle, the portfolio implications depend on the risk characteristics of real estate as an asset class. [Adelino, Schoar, and Severino \(2018\)](#) find that a large majority of U.S. households (71%) believe that housing is a “safe” investment. When housing is considered a safe asset, we may expect the implications for portfolio choice to go against the findings in this paper: in a homothetic model, the net effect of changes to financial wealth and human capital should be negative instead of zero, since compositional shifts away from a safe outside investment should – analogous to the case with bond-like human capital – lead to a reduction of the equity share in the riskier financial portfolio to get the same overall risk exposure.

As an additional check, I look at the portfolios of home owners versus renters in the SCF. [Figure A.2](#) plots the equity share over the life cycle for these two groups. Home owners have a slightly higher equity share than renters, with a similar age profile. If housing were treated as a

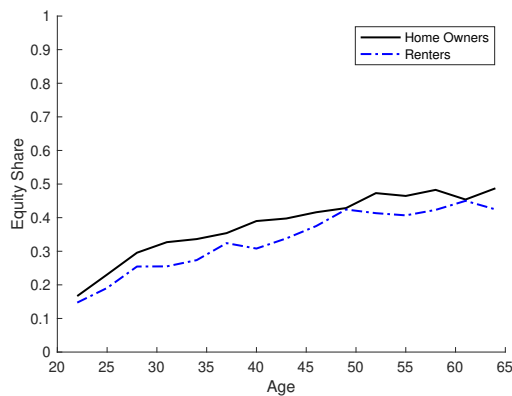
Table A.1: Wealth Composition of RIs in SCF

Variable	Mean	SD	P10	P25	P50	P75	P90
Share of investable wealth							
Retirement wealth	0.88	0.24	0.48	0.88	1	1	1
Share of net worth							
Retirement wealth	0.51	0.94	0.08	0.19	0.40	0.63	0.88
Non-retirement investable wealth	0.07	0.26	0	0	0	0.05	0.23
Home equity	0.37	0.85	0	0.03	0.27	0.51	0.76
Liquid wealth	0.10	0.27	0.01	0.02	0.04	0.11	0.24
Other	-0.05	1.61	-0.30	-0.01	0.07	0.22	0.47

*Notes:* This table summarizes the composition of net worth of retirement investors in the 2016 Survey of Consumer Finances. The sample consists of households of age 30–58 with wage income above the minimum wage income, with net worth of at least one thousand dollars, and with retirement wealth that falls within the 10th and 90th percentiles of the retirement wealth distribution by age. Investable wealth is defined as the total of money market funds, non-money market funds, individual stocks and bonds, certificate of deposits, quasi-liquid retirement wealth, and other managed accounts.

risky investment, larger housing wealth should lead to a reduction in the equity share of financial wealth, everything else equal. In contrast, the observed patterns are consistent with a wealth effect on risk tolerance. Finally, including yearly zip code fixed effects in the portfolio regressions accounts for any correlations between local changes in house prices and changes in labor income or asset returns. The measured portfolio changes in response to wealth changes do not change when including zip code–year fixed effects.

Figure A.2: Equity Share over the Life Cycle by Home Ownership in SCF



*Notes:* This figure plots the age profile of equity shares for investors with retirement wealth in the SCF, comparing home owners and renters.

Table A.2: Income Growth and Equity Share Changes – Job Spells

	$\Delta_{3+j} \log \text{equity share}_{t+j}$							
	$j = 0$	$j = 1$	$j = 3$	$j = 5$	$j = 0$	$j = 1$	$j = 3$	$j = 5$
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$\Delta_3 \log \text{income}_t$	0.0194 (0.0012)	0.0236 (0.0013)	0.0251 (0.0016)	0.0213 (0.0027)	0.0383 (0.0063)	0.0327 (0.0062)	0.0236 (0.0073)	0.0081 (0.0133)
Employer $\times$ year FE	Y	Y	Y	Y				
Log equity share $_{t-3}$	Y	Y	Y	Y	Y	Y	Y	Y
Demographic controls	Y	Y	Y	Y	Y	Y	Y	Y
Year FE					Y	Y	Y	Y
Sample	Same job in $t - 3$ and $t$				Job change between $t - 3$ and $t - 1$			
R-squared	0.264	0.288	0.328	0.371	0.450	0.481	0.513	0.523
Share of individuals	49.2%	47.7%	40.9%	25.2%	2.3%	2.3%	1.8%	0.7%

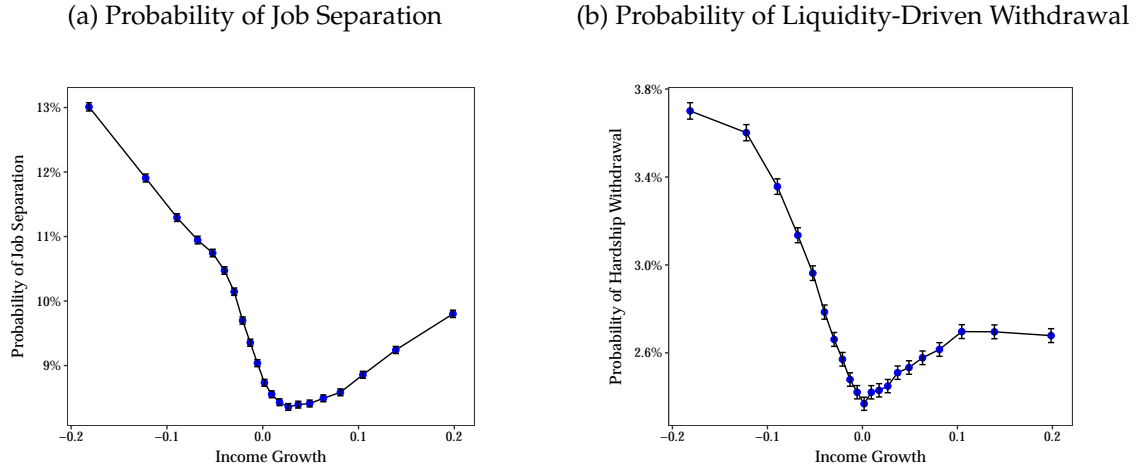
*Notes:* This table presents the results of an OLS regression of changes in log equity shares, measured over several horizons, on three-year income growth. Columns (1)–(4) report the results for the subsample of investors with the same job in  $t - 3$  and  $t$ . Columns (5)–(8) report the results for the subsample of investors that had a job change between  $t - 3$  and  $t - 1$ . The demographic controls include a second-order polynomial in age, gender, marital status, a second-order polynomial in employment tenure, log income, and the log of financial assets, all measured at  $t - 3$ . Standard errors are clustered at the individual level.

### A.3.2 Job Turnover and Background Risk

Most individuals have a single job over the period that they are in the sample. As a result, the results in Section 4 on the relation between income growth and changes in equity shares are largely driven by within-employment changes in income. The first four columns in Table A.2 make this selection explicit by selecting the sample to be investors with a single job during the window of portfolio changes. For robustness, I repeat the analysis for the relatively smaller subsample of individuals (but still sizeable in numbers) that switched jobs in between income observations. The results are in columns (5)–(8). For this subsample, I find a positive and significant effect of income growth on changes in equity share that is consistent with the baseline estimates. The magnitude of the effect is even somewhat larger, perhaps because the relative share of permanent income growth in total income growth is larger when comparing income across jobs. For this subsample the point estimate declines with the horizon, although power is limited at long horizons.

Next, in models of individual financial decisions, the risk properties of human capital are crucial for determining optimal allocations of investable wealth. Because human capital is non-tradable, not just systematic risk but also uninsurable wage risk matters for portfolio allocations. The quantitative importance of uninsurable wage risk was recently studied by [Fagereng et al. \(2018\)](#). By instrumenting variation in worker earnings by variation in firm profitability, they find a large marginal effect of uninsurable income risk on portfolio choices. In standard models of wage income, there is no relation between individual income growth and changes to future uninsurable wage risk. However, if income growth is in fact negatively related to future variability of earnings, that would provide an alternative explanation for the positive relation between income growth

Figure A.3: Background Risk and Income Growth



Notes: Panel (a) plots the probability of having a job separation in year  $t + 1$  as a function of income growth in year  $t$ , using 20 bins for income growth. Panel (b) plots the probability of having a liquidity-driven withdrawal in year  $t + 1$  as a function of income growth in year  $t$ , again using 20 bins for income growth.

and equity share changes.

Lacking a direct measure of background risk in the data, I consider the probability of job separation as a proxy. I construct an indicator for job separation in year  $t + 1$  for individuals that have an active employment relation at the end of year  $t$  that equals one if the employment relation has been terminated by the end of year  $t + 1$ . Figure A.3a plots the probability of having a job separation in year  $t + 1$  as a function of income growth in year  $t$ , using 20 bins for income growth. Consistent with Holzheu (2018), there is a U-shaped relationship between income growth and job separations. Workers with high negative or positive changes in wages have a higher probability of job separation in the next period. To confirm that large changes in earnings represent a real economic risk for households, especially on the right tail of the income growth distribution, I also measure withdrawals from retirement accounts due to liquidity needs. Figure A.3b plots the probability of having such a withdrawal in year  $t + 1$  as a function of income growth in year  $t$ , again using 20 bins for income growth. The result is a similar U-shaped relation.

If background risk is the channel through which income growth affects portfolio allocations, we would expect a similar nonlinear relation between income growth and equity share changes. In Table A.3, I test this prediction. In addition to the basic set of controls from the portfolio regression and various fixed effects, I control for mean reversion in portfolio shares with a strength that increases in the magnitude of income growth (see Figure A.8) by including the interaction between absolute income growth and initial equity share as a control. Columns (1)–(4) confirm a positive relation between squared income growth and separations that holds after including the controls. Next, I add squared income growth to the portfolio regression. The results are in columns (5)–(8). The coefficient on squared income growth is a precisely estimated zero in all specifications.

Table A.3: Income Growth and Future Income Risk

	Job separation <sub>t+1</sub>				$\Delta \log$ equity share <sub>t</sub>			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$\Delta \log$ income <sub>t</sub>	-0.1303 (0.0008)	-0.1305 (0.0008)	-0.1228 (0.0008)	-0.1288 (0.0009)	0.0184 (0.0009)	0.0146 (0.0009)	0.0139 (0.0009)	0.0142 (0.0010)
$(\Delta \log$ income <sub>t</sub> ) <sup>2</sup>	0.1883 (0.0182)	0.1367 (0.0182)	0.1987 (0.0178)	0.1869 (0.0181)	-0.0015 (0.0204)	-0.0144 (0.0204)	-0.0269 (0.0204)	-0.0366 (0.0208)
Log equity share <sub>t-1</sub>	-0.0020 (0.0002)	-0.0010 (0.0002)	0.0004 (0.0002)	0.0004 (0.0002)	-0.2352 (0.0009)	-0.2359 (0.0009)	-0.2356 (0.0009)	-0.2355 (0.0009)
$ \Delta \log$ income <sub>t</sub>	0.0665 (0.0036)	0.0749 (0.0036)	0.0419 (0.0036)	0.0423 (0.0036)	-0.0443 (0.0041)	-0.0394 (0.0041)	-0.0357 (0.0041)	-0.0330 (0.0042)
$ \Delta \log$ income <sub>t</sub>   × log equity share <sub>t-1</sub>	-0.0035 (0.0020)	-0.0052 (0.0020)	-0.0070 (0.0019)	-0.0072 (0.0019)	-0.0909 (0.0092)	-0.0926 (0.0092)	-0.0982 (0.0090)	-0.0975 (0.0091)
Year FE	Y				Y			
Demographic controls	Y	Y	Y	Y	Y	Y	Y	Y
Demographic controls × log equity share <sub>t-1</sub>	Y	Y	Y	Y	Y	Y	Y	Y
Industry × year FE		Y				Y		
Employer × year FE			Y				Y	
Employer × income bin × year FE				Y				Y
R-squared	0.016	0.026	0.104	0.125	0.153	0.156	0.180	0.193
Share of individuals	92.3%	91.8%	92.3%	91.9%	92.7%	92.3%	92.7%	92.4%

*Notes:* This table reports OLS estimates of two outcome variables on a second-order polynomial in income growth. Columns (1)–(4) report estimates of the relation between job separation rates and quadratic income growth. Columns (5)–(8) report estimates of the relation between changes in log equity shares and quadratic income growth. The demographic controls include a second-order polynomial in age, gender, marital status, a second-order polynomial in employment tenure, log income, and the log of financial assets, all measured at  $t - 1$ . Standard errors are clustered at the individual level.

Hence, I find no evidence that income growth affects portfolio risk taking through background risk instead of a first-order wealth effect.

### A.3.3 Heterogeneity in Effect Size across Investors and Accounts

Next, I test for sources of heterogeneity in the relation between income growth and portfolio equity changes across the population. I consider heterogeneity across investors at different stages of the life cycle and with different financial profiles, and further break down the results by the magnitude of income changes.

**Life-Cycle Variation** To explore heterogeneity in the relation between income growth and portfolio equity share changes by demographic characteristics, I test for variation in the effect size by age, financial assets, and income. Table A.4 reports the results. I find that the relation between income growth and equity share changes is larger for older investors. There is little variation in the effect by total asset wealth and by the level of income. Since the effects of income growth on



Table A.4: Income Growth and Equity Share Changes – Heterogeneity by Investor Characteristics

	$\Delta \log \text{equity share}_t$					
	(1)	(2)	(3)	(4)	(5)	(6)
$\Delta \log \text{income}_t$	0.0253	0.0740				
$\times \text{Age}_{t-1} < 40$	(0.0016)	(0.0109)				
$\Delta \log \text{income}_t$	0.0440	0.1541				
$\times \text{Age}_{t-1} \in [40, 49]$	(0.0022)	(0.0131)				
$\Delta \log \text{income}_t$	0.0661	0.2482				
$\times \text{Age}_{t-1} \geq 50$	(0.0034)	(0.0174)				
$\Delta \log \text{income}_t$			0.0283	0.1354		
$\times \text{Assets}_{t-1} \in \text{bottom tercile}$			(0.0019)	(0.0152)		
$\Delta \log \text{income}_t$			0.0462	0.1978		
$\times \text{Assets}_{t-1} \in \text{middle tercile}$			(0.0023)	(0.0150)		
$\Delta \log \text{income}_t$			0.0492	0.1390		
$\times \text{Assets}_{t-1} \in \text{top tercile}$			(0.0026)	(0.0116)		
$\Delta \log \text{income}_t$					0.0493	0.2064
$\times \text{Income}_{t-2} \in \text{bottom tercile}$					(0.0022)	(0.0180)
$\Delta \log \text{income}_t$					0.0452	0.2295
$\times \text{Income}_{t-2} \in \text{middle tercile}$					(0.0025)	(0.0177)
$\Delta \log \text{income}_t$					0.0445	0.1372
$\times \text{Income}_{t-2} \in \text{top tercile}$					(0.0023)	(0.0105)
Year FE	Y	Y	Y	Y	Y	Y
Log equity share $_{t-1}$	Y	Y	Y	Y	Y	Y
Demographic controls	Y	Y	Y	Y	Y	Y
Income instrumented	Y	Y	Y	Y	Y	Y
Sample		Active turnover		Active turnover		Active turnover
R-squared	0.144	0.336	0.149	0.337	0.149	0.337
Share of individuals	96.7%	23.0%	96.7%	23.0%	96.7%	23.0%

*Notes:* This table presents the results of an IV regression of one-year changes in log equity shares on income growth interacted by various indicators. The demographic controls include a second-order polynomial in age, gender, marital status, a second-order polynomial in employment tenure, log income, and the log of financial assets, all measured at  $t - 1$ . Log income at  $t - 1$  and  $t$  is instrumented by log income at  $t - 2$  and  $t + 1$ , respectively. Standard errors are clustered at the individual level.

portfolio changes are driven by a small set of people that make substantial changes, I also report the estimates conditional on active turnover over the year (defined as a turnover of more than 25% of initial assets). In this way, we can rule out that differences by demographics are driven by a differential likelihood of portfolio adjustment. The patterns are similar when conditioning on active turnover.

**Small versus Large Changes in Labor Income** Theories that generate DRRA preferences differ in their predictions on heterogeneity in the effect by shock size: a habit specification predicts that big changes have the largest effects, while models with consumption commitments predict a concentration of the effect on small shocks. Table A.5 reports the regression results when restricting the sample to various ranges of income growth. For the OLS specification from

Table A.5: Income Growth and Equity Share Changes – Magnitude of Income Change

	$\Delta \log \text{ equity share}_t$							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$\Delta \log \text{ income}_t$	0.0040 (0.0004)	0.0163 (0.0009)	0.0301 (0.0017)	0.0402 (0.0036)	0.0417 (0.0013)	0.0428 (0.0017)	0.0538 (0.0026)	0.0667 (0.0063)
Year FE	Y	Y	Y	Y	Y	Y	Y	Y
Log equity share $_{t-1}$	Y	Y	Y	Y	Y	Y	Y	Y
Demographic controls	Y	Y	Y	Y	Y	Y	Y	Y
Income instrumented	N	N	N	N	Y	Y	Y	Y
$ \Delta \log \text{ income}_t $ in range	$[0, \infty)$	$[0, 0.25]$	$[0, 0.1]$	$[0, 0.05]$	$[0, \infty)$	$[0, 0.5]$	$[0, 0.25]$	$[0, 0.1]$
R-squared	0.144	0.142	0.140	0.139	0.144	0.143	0.142	0.141
Share of individuals	96.7%	93.0%	83.7%	70.4%	96.7%	92.0%	85.1%	67.9%

*Notes:* This table presents regression estimates of one-year changes in log equity shares on income growth for various restrictions on the range of income growth. Columns (1)–(4) report the results for the OLS specification. Columns (5)–(8) report the results for the IV specification where log income at  $t - 1$  and  $t$  is instrumented by log income at  $t - 2$  and  $t + 1$ , respectively. The demographic controls include a second-order polynomial in age, gender, marital status, a second-order polynomial in employment tenure, log income, and the log of financial assets, all measured at  $t - 1$ . Standard errors are clustered at the individual level.

Section 4.1, the magnitude of the effect strongly increases as the range of income growth gets narrowed down. For the IV specification, the magnitude still increases when restricting the sample to smaller shocks, but the relative differences are much smaller.

### A.3.4 Other Portfolio Outcomes

The results in the main text concentrate on equity share as the measure that summarizes investor portfolios. Here, I show the effects of income growth on other portfolio outcomes.

Table A.6 reports results for the main IV specification applied to other portfolio measures. Column (1) has the market beta of the portfolio as outcome variable. The results closely match the findings for the equity share (column (5) of Table 4). The reason is that the dominating source of variation across investor portfolios is variation in holdings across asset classes. There is much less variation in market exposure within asset classes. The second column shows that income growth does not lead to an economically meaningful increase in the market beta of equity. Instead, income growth leads to an increase in the market exposure of investor portfolios by a reallocation from both fixed income and cash-like securities to equity products, as evident from columns (3) and (4).

Columns (5)–(7) of Table A.6 display the results for three other portfolio measures. Column (5) shows that the main effect of income growth on portfolio equity shares is driven in part by a positive relation between income growth and the share of assets invested in individual stocks. There is no effect on the share of equity invested in international equity funds or securities (column 6). Finally, I find no relation between income growth and the share of assets invested in target date

Table A.6: Income Growth and Portfolio Changes

	$\Delta$ portfolio measure <sub><i>t</i></sub>						
	Portfolio beta	Equity beta	Fixed income share	Cash share	Indiv stock share	Intl share of equity	TDF share
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
$\Delta \log \text{income}_t$	0.0248 (0.0005)	0.0015 (0.0003)	-0.0053 (0.0004)	-0.0173 (0.0004)	0.0096 (0.0003)	0.0005 (0.0003)	-0.0003 (0.0006)
Year FE	Y	Y	Y	Y	Y	Y	Y
Portfolio measure <sub><i>t-1</i></sub>	Y	Y	Y	Y	Y	Y	Y
Demographic controls	Y	Y	Y	Y	Y	Y	Y
Income instrumented	Y	Y	Y	Y	Y	Y	Y
R-squared	0.110	0.088	0.114	0.096	0.071	0.090	0.031
Share of individuals	99.7%	96.6%	100.0%	100.0%	100.0%	96.7%	100.0%

*Notes:* This table presents the results of an IV regression of one-year changes in various portfolio outcomes on income growth. The demographic controls include a second-order polynomial in age, gender, marital status, a second-order polynomial in employment tenure, log income, and the log of financial assets, all measured at  $t - 1$ . Log income at  $t - 1$  and  $t$  is instrumented by log income at  $t - 2$  and  $t + 1$ , respectively. Standard errors are clustered at the individual level.

funds (column 7). This last finding suggests that the positive effect of income growth on equity shares reflects investor preferences and cannot be explained by investors moving out of default allocations or moving into more “advised” products like target date funds for other reasons.

### A.3.5 Active Rebalancing Decisions and Heterogeneity in Propensity to Rebalance

Finally, I present additional results related to rebalancing decisions and differences in rebalancing propensities across investors.

First, column (1) of Table A.7 presents estimates of the partial adjustment regression model with equity shares in levels instead of in logs. The results are consistent and are therefore not driven by the specific functional forms that separate passive changes from realized overall portfolio returns.

Second, column (2) reports the effect of income growth and idiosyncratic portfolio returns on changes in equity shares in years where the individual has an active portfolio turnover (turnover above 25% of initial assets over the year). Consistent with the findings in Section 4.3, both coefficients on income growth and portfolio returns are much larger in absolute value for the set of people that make an active reallocation decision. While in general portfolio returns and changes in equity shares are positively correlated due to passivity, these results confirm that investors actively offset a positive idiosyncratic portfolio return by slightly reducing the portfolio equity share.

Third, there is considerable heterogeneity in the propensity to rebalance portfolios across investors: Table A.8 shows that individual fixed effects explain around half of the overall variation in rebalancing. As illustrated in Section 5.1, rebalancing is hard to predict based on changes in individual financial situations. Since the timing of rebalancing is (nearly) unrelated to

Table A.7: Portfolio Responses Conditional on Rebalancing

	$\Delta$ equity share <sub><i>t</i></sub>				
	(1)	(2)	(3)	(4)	(5)
Passive change in equity share <sub><i>t</i></sub>	0.8641 (0.0029)				
$\Delta$ log income <sub><i>t</i></sub>	0.0245 (0.0007)	0.0693 (0.0034)	0.0633 (0.0036)	0.0462 (0.0047)	0.0940 (0.0056)
Log portfolio return <sub><i>t</i></sub>	-0.0041 (0.0008)	-0.0152 (0.0039)	-0.0108 (0.0043)	-0.0299 (0.0058)	0.0046 (0.0062)
Year FE	Y	Y	Y	Y	Y
Log equity share <sub><i>t-1</i></sub> (3rd order) × year FE	Y	Y	Y	Y	Y
Demographic controls	Y	Y	Y	Y	Y
Income instrumented	Y	Y	Y	Y	Y
Sample		Active turnover	Active turnover, >= 3 years	Active turnover, >= 3 years, frequent rebalancing	Active turnover, >= 3 years, infrequent rebalancing
R-squared	0.166	0.341	0.337	0.323	0.369
Share of individuals	56.6%	16.9%	13.5%	5.2%	8.3%

*Notes:* The first column presents regression estimates of the partial adjustment model with equity shares in levels. The second column considers portfolio adjustment for investors with active turnover (a turnover of at least 25% of initial assets) over the year. The third column restricts the sample to investors with observations for at least three years and that have active turnover over the year. The fourth and fifth columns further split the sample into people with active portfolio turnover in more than one third of the total years in the sample, and people with a large portfolio turnover in at most one third of the total years in the sample. The demographic controls include a second-order polynomial in age, gender, marital status, a second-order polynomial in employment tenure, log income, and the log of financial assets, all measured at  $t - 1$ . Log income at  $t - 1$  and  $t$  is instrumented by log income at  $t - 2$  and  $t + 1$ , respectively. Standard errors are clustered at the individual level.

observable economic fundamentals, the investment decisions of investors that actively rebalance their portfolios are informative about the overall effects of wealth changes on risk preferences. If population behavior is homogeneous, as assumed in the baseline specification, the effect on desired portfolio allocations follows from scaling the average change in portfolios by the average propensity to rebalance. However, some investors never update their portfolios, while other investors frequently update their portfolios.

A potential issue of the baseline specification is therefore that people that differ in their rebalancing behavior might also differ in their sensitivity to wealth changes. For instance, if investors that update their portfolios regularly are more sensitive to wealth changes, the overall elasticity of desired portfolio allocations to wealth fluctuations is overestimated. In columns (3)–(5) of Table A.7, I estimate the portfolio responses to income growth and idiosyncratic portfolio

Table A.8: Trading Behavior

	Trade indicator				Active turnover indicator			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$ \Delta \log \text{income}_t $		0.0260 (0.0025)		0.0383 (0.0048)		0.0294 (0.0019)		0.0212 (0.0041)
$ \text{Log equity return}_t $		0.2713 (0.0025)		0.1010 (0.0044)		0.2045 (0.0020)		0.0828 (0.0038)
Year FE	Y	Y	Y	Y	Y	Y	Y	Y
Demographic controls	Y	Y	Y	Y	Y	Y	Y	Y
Individual FE			Y	Y			Y	Y
Sample			Balanced	Balanced			Balanced	Balanced
R-squared	0.061	0.062	0.540	0.540	0.022	0.023	0.391	0.391
Share of individuals	54.1%	54.1%	6.2%	6.2%	54.1%	54.1%	6.2%	6.2%

*Notes:* This table presents regression estimates of two measures of portfolio reallocation activity on the magnitude of income growth changes and realized equity returns of individual portfolios. In columns (1)–(4), the outcome variable is an indicator for having at least one investor-driven trade during the year. In columns (5)–(8), the outcome variable is an indicator for having a turnover of at least 25% of initial assets over the year. The demographic controls include a second-order polynomial in age, gender, marital status, a second-order polynomial in employment tenure, log income, and the log of financial assets, all measured at  $t - 1$ . Standard errors are clustered at the individual level.

returns for a sample of investors with observations for at least three years. Column (3) confirms that the results for this subsample are similar to the full sample. Columns (4) and (5) further split the sample into people that frequently rebalance their portfolios (active portfolio turnover in more than one third of the total years in the sample) and people that infrequently rebalance their portfolios (active portfolio turnover in at most one third of the total years in the sample). The qualitative findings are consistent across these populations, and in fact the portfolio results are stronger for people that irregularly rebalance their portfolios. Conditional on rebalancing, people that frequently rebalance are more sensitive to portfolio returns, but people that infrequently rebalance are more sensitive to income growth.

## A.4 Model Appendix

### A.4.1 Numerical Procedure

**Model solution.** Following the usual approach in solving life-cycle models, I start from the last period of life and solve the life-cycle problem backwards. Here, I describe the details of the solution method. For notational simplicity, I suppress the individual subscript  $i$ .

For computing the value function and optimal policies, I scale all relevant variables by permanent income  $G_t P_t$ : let  $v_t \equiv V_t / (G_t P_t)$ ,  $w_t \equiv W_t / (G_t P_t)$ ,  $c_t \equiv C_t / (G_t P_t)$ ,  $y_t^{post} \equiv Y_t^{post} / (G_t P_t)$ .

The state variables are  $w_t, P_t$ , and  $\tilde{\theta}_t$ , where

$$\tilde{\theta}_t = \begin{cases} \theta_t^{pass} & \text{if no portfolio update at } t, \text{ with probability } 1 - \chi \\ -1 & \text{if portfolio update at } t, \text{ with probability } \chi. \end{cases} \quad (\text{A.4.1})$$

By definition, the portfolio share  $\theta_t$  equals  $\tilde{\theta}_t$  if there is no portfolio update at  $t$ , which happens with probability  $1 - \chi$ . With probability  $\chi$  the investor chooses a new portfolio allocation, in which case the history of portfolio shares is irrelevant.

The value function in period  $t \leq T_D$  is<sup>34</sup>

$$v_t = \max_{c_t, \theta_t} \left\{ (1 - \beta)c_t^{1 - \frac{1}{\psi}} + \beta(G_t P_t)^{\frac{1}{\psi} - 1} \left\{ J^{-1} \left( \mathbb{E}_t \left[ \pi_t J(G_{t+1} P_{t+1} v_{t+1}) + (1 - \pi_t) J \left( \frac{G_{t+1} P_{t+1} w_{t+1}}{\tilde{b}} \right) \right] \right) \right\}^{1 - \frac{1}{\psi}} \right\}^{\frac{1}{1 - \frac{1}{\psi}}}, \quad (\text{A.4.2})$$

subject to

$$w_{t+1} = (w_t - c_t)(R^f + \theta_t(R_{t+1}^e - R^f)) \frac{G_t P_t}{G_{t+1} P_{t+1}} + y_{t+1}^{post} \quad (\text{A.4.3})$$

$$\theta_t \begin{cases} = \tilde{\theta}_t & \text{if } \tilde{\theta}_t \geq 0 \\ \in [0, 1] & \text{if } \tilde{\theta}_t = -1. \end{cases} \quad (\text{A.4.4})$$

The first-order conditions for asset allocation and consumption, respectively, are given by

$$\begin{aligned} 0 &= \mathbb{E}_t \left[ \left( \pi_t J'(G_{t+1} P_{t+1} v_{t+1}) \frac{\partial v_{t+1}}{\partial w_{t+1}} + (1 - \pi_t) J' \left( \frac{G_{t+1} P_{t+1} w_{t+1}}{\tilde{b}} \right) \frac{1}{\tilde{b}} \right) (w_t - c_t)(R_{t+1}^e - R^f) + \right. \\ &\quad \left. \pi_t J'(G_{t+1} P_{t+1} v_{t+1}) \frac{G_{t+1} P_{t+1}}{G_t P_t} \frac{\partial v_{t+1}}{\partial \tilde{\theta}_{t+1}} \frac{R_{t+1}^e R^f}{(R^f + \theta_t(R_{t+1}^e - R^f))^2} \mathbb{1}(\tilde{\theta}_{t+1} \geq 0) \right] \\ c_t^{-\frac{1}{\psi}} &= \frac{\beta}{1 - \beta} (G_t P_t)^{\frac{1}{\psi}} \left\{ J^{-1} \left( \mathbb{E}_t \left[ \pi_t J(G_{t+1} P_{t+1} v_{t+1}) + (1 - \pi_t) J \left( \frac{G_{t+1} P_{t+1} w_{t+1}}{\tilde{b}} \right) \right] \right) \right\}^{-\frac{1}{\psi}} \\ &\quad \times (J^{-1})' \left( \mathbb{E}_t \left[ \pi_t J(G_{t+1} P_{t+1} v_{t+1}) + (1 - \pi_t) J \left( \frac{G_{t+1} P_{t+1} w_{t+1}}{\tilde{b}} \right) \right] \right) \\ &\quad \times \mathbb{E}_t \left[ \left( \pi_t J'(G_{t+1} P_{t+1} v_{t+1}) \frac{\partial v_{t+1}}{\partial w_{t+1}} + (1 - \pi_t) J' \left( \frac{G_{t+1} P_{t+1} w_{t+1}}{\tilde{b}} \right) \frac{1}{\tilde{b}} \right) R_{t+1}^p \right], \end{aligned} \quad (\text{A.4.5})$$

where  $R_{t+1}^p \equiv R^f + \theta_t(R_{t+1}^e - R^f)$ . The life-cycle problem is solved iteratively by calculating the solution to these FOCs, accounting for the constraints, and calculating the corresponding value function.

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<sup>34</sup>Note that  $\pi_{T_D} = 0$  and therefore  $v_t = \max_{c_t, \theta_t} \left\{ (1 - \beta)c_t^{1 - \frac{1}{\psi}} + \beta(G_t P_t)^{\frac{1}{\psi} - 1} \left\{ J^{-1} \left( \mathbb{E}_t \left[ J \left( \frac{G_{t+1} P_{t+1} w_{t+1}}{\tilde{b}} \right) \right] \right) \right\}^{1 - \frac{1}{\psi}} \right\}^{\frac{1}{1 - \frac{1}{\psi}}}$  for  $t = T_D$ .

To solve the model numerically, I construct a grid for the state variables. I discretize wealth-to-income  $w$  (40 points), permanent income  $P$  (40 grid points), and the existing portfolio allocation  $\tilde{\theta}$  (12 grid points). The grid points for  $w$  and  $P$  are equally spaced on a logarithmic scale, and the grid points for  $\tilde{\theta}$  are equally spaced on a linear scale. For computational efficiency, I use the endogenous grid point method and specify a grid for post-consumption total asset holdings  $n$  (20 grid points). Given values for  $n$  and  $P$ , I solve for the optimal allocation decision conditional on portfolio adjustment. Next, I use the consumption FOC to calculate the optimal consumption corresponding to the given values for  $n$  and  $\theta$ . Finally, I interpolate on the grid to get consumption and portfolio choice for exogenous grid points  $w$ . I use linear interpolation and extrapolation to evaluate the value function outside of the grid points. Because of Epstein-Zin preferences, the value function is close to linear in cash on hand, except when wealth is low and close to the borrowing constraint, and flat in  $P$  with CRRA preferences and proportional government policies. Expectations are evaluated through Gaussian quadrature.

**Estimation.** I estimate the parameters of the model by minimizing the distance between the moments in model simulations and the corresponding moments in the data. As weighting matrix  $W$ , I use the inverse of the diagonal of the covariance matrix  $\Sigma$  of the empirical moments. I calculate the value function and policy functions for a four-dimensional grid of the parameters  $\gamma_0, \gamma_1, \chi$ , and  $\beta$  for a total of 3000 points. I then simulate the model for different values of  $\beta_0$  and  $\beta_1$ . I use cubic splines to approximate the objective function outside of the grid points. The estimated parameter vector is given by the minimum of the approximated objective function.

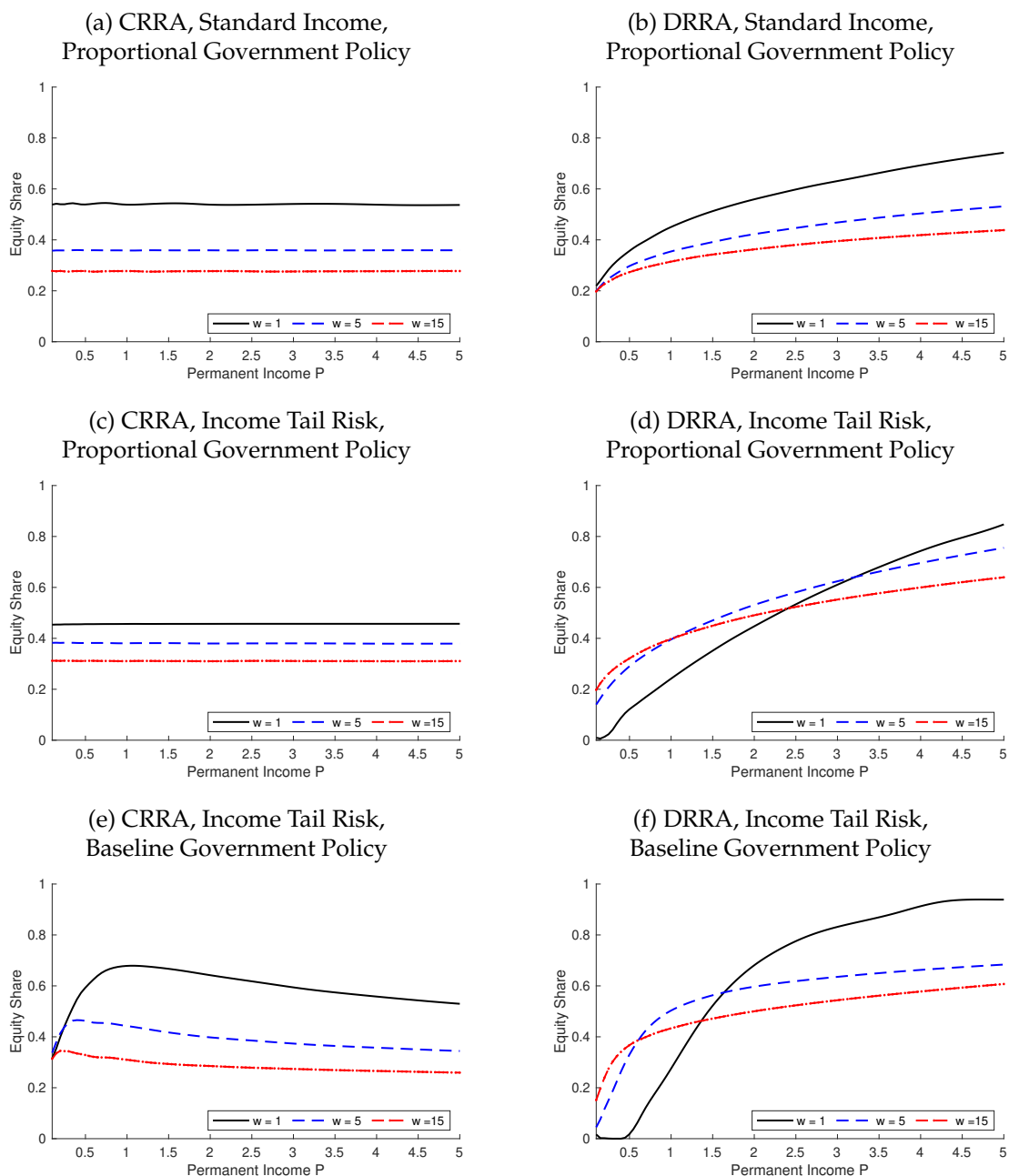
To calculate standard errors, I compute the Jacobian matrix  $D$  of the moments with respect to the parameters. The standard errors of the estimated parameters are given by the square roots of the diagonal elements of the matrix  $\Omega$ , which is defined as:

$$\Omega = (D'WD)^{-1}D'W\Sigma WD'(D'WD)^{-1}. \quad (\text{A.4.6})$$

#### A.4.2 Policy Functions

The optimal policies for consumption and asset allocations in the model are functions of the state variables  $w_{it}$ ,  $P_{it}$ , and  $\tilde{\theta}_{it}$ . To illustrate optimal asset allocations conditional on updating the portfolio, Figure A.4 plots the equity share policy function at age 50, under the estimated parameter values, for six cases of the model: CRRA versus DRRA preferences and normal income shocks versus income with countercyclical tail risk in the model with proportional taxes and transfers, and CRRA versus DRRA preferences with countercyclical tail risk, progressive taxes, and income-dependent retirement replacement rates. The figures plot the optimal  $\theta$  as a function of  $P$ , for given values of normalized cash-on-hand  $w$ , and conditional on having the opportunity of updating the portfolio.

Figure A.4: Optimal Allocation Policy in Model



Notes: This figure plots the policy function for the portfolio equity share  $\theta_{it}$  at age 50 as a function of permanent income  $P$ , for different values of cash on hand relative to permanent income  $w$  and conditional on having the opportunity to update the portfolio. The six panels cover different cases of the model: CRRA versus DRRA preferences, income with normal shocks versus income with countercyclical tail risk, and proportional taxes and transfers versus progressive taxes and income-dependent retirement replacement rates.

The policy functions highlight two key channels in the effects of wealth on portfolio choice. The first channel is the role of human capital in the composition of total wealth. The relation



between relative cash on hand and optimal portfolio shares depends on the properties of human capital. When income is relatively stable with a low correlation to stock returns, as in the standard income specification, human capital serves as a substitute for bonds. As a result, the optimal allocation of financial wealth to stocks strongly decreases with relative cash on hand in panel (a). With countercyclical tail risk in labor income, income is substantially riskier and less bond-like. As a result, the optimal equity share is only mildly decreasing in relative cash on hand in panel (c).

The second key channel is the effect of overall wealth on risk aversion. In panels (a) and (c), the optimal equity allocation is flat in  $P$  for a fixed value of  $w$ , since the value function is homothetic in permanent income. In contrast, panels (b) and (d) illustrate the policy function under non-homotheticity in risk preferences. For a fixed value of relative cash on hand, the equity share increases with permanent income. This reflects the effect of decreasing relative risk aversion in total wealth. Panel (e) shows that the homotheticity of the value function is broken by non-proportional taxes and transfers, in which case the optimal allocation policy depends on the level of wealth even with CRRA preferences. Panel (f) shows a similar relation under DRRA preferences between risk-taking behavior and permanent income in the baseline specification compared to panel (d) with proportional government policies, with a higher sensitivity of equity shares to permanent income at low levels of financial wealth.

### A.4.3 Robustness and Parameter Sensitivity

**Variations to baseline setup.** Table A.9 shows that results are robust to changes in the model and estimation setup. Column (1) repeats the results of the baseline model estimation. Column (2) shows that these results are robust to using a different weighting matrix: the inverse of the full covariance matrix of the empirical moments. Columns (3)–(6) present the results for alternative calibrated parameter values. Column (2) fixes the EIS at  $\psi = 0.2$  (instead of 0.5) and column (3) fixes the EIS at  $\psi = 0.8$ . Choosing a different EIS has strong implications for the heterogeneity in discount rates  $\beta_1$  that is needed to match cross-sectional variation in saving rates, but does not materially affect the estimates of the other parameters.

Columns (4), (5), and (6) do not show any meaningful differences compared to the baseline specification when either choosing a higher EIS ( $\psi = 0.8$ ), setting a lower weight on bequests ( $\tilde{b} = 0.5$ ), or using the estimated income profile from high school educated households instead of college-educated households from Cocco et al. (2005).

**Sensitivity.** Figure A.5 plots the sensitivity matrix defined in Andrews et al. (2017). The values are scaled to correspond to a one standard error increase in each moment.

*Baseline risk aversion*  $\gamma_0$ . The baseline coefficient of risk aversion is mainly driven by the average equity share at age 50 (age group 49). A higher average equity share leads to a lower average risk aversion. Higher saving rates somewhat increase risk aversion, due to the precautionary motive, and more positive responses to wealth shocks imply a lower risk aversion.

Table A.9: Parameter Estimates in Alternative Model Specifications

	Baseline	Full weighting matrix	$\psi = 0.2$	$\psi = 0.8$	$\tilde{b} = 0.5$	High school income profile	Data
	(1)	(2)	(3)	(4)	(5)	(6)	
Baseline risk aversion $\gamma_0$	5.7118 (0.0786)	5.7431 (0.0778)	5.7117 (0.0804)	5.6623 (0.0768)	5.6876 (0.0767)	5.6897 (0.0746)	
Non-homotheticity in risk aversion $\gamma_1$	0.3106 (0.0158)	0.3024 (0.0149)	0.2587 (0.0159)	0.3060 (0.0153)	0.3134 (0.0156)	0.3141 (0.0149)	
Calvo probability $\chi$	0.1520 (0.0039)	0.1524 (0.0038)	0.1648 (0.0045)	0.1520 (0.0038)	0.1504 (0.0039)	0.1628 (0.0045)	
Baseline discount rate $\beta_0$	0.9482 (0.0017)	0.9499 (0.0016)	0.9503 (0.0034)	0.9485 (0.0012)	0.9509 (0.0018)	0.9477 (0.0017)	
Slope of discount rate $\beta_1$	4.2358 (0.3632)	4.0564 (0.3718)	24.1113 (2.6631)	2.0645 (0.1891)	5.5289 (0.4331)	4.4391 (0.3437)	
Income process	Tail risk	Tail risk	Tail risk	Tail risk	Tail risk	Tail risk	
Targets							
Age profile of financial wealth	Y	Y	Y	Y	Y	Y	
Equity share at age 50	Y	Y	Y	Y	Y	Y	
Portfolio regression $b_0$	Y	Y	Y	Y	Y	Y	
Portfolio regression $b_1, b_2$	Y	Y	Y	Y	Y	Y	
Saving regression $\phi$	Y	Y	Y	Y	Y	Y	
Value of objective	50.880	40.081	22.021	62.683	66.801	73.944	
Equity share at age 50	0.4170	0.4171	0.4143	0.4328	0.4184	0.4295	0.4309 (0.0111)
Portfolio regression $b_0$	0.8646	0.8637	0.8520	0.8648	0.8672	0.8574	0.8372 (0.0059)
Portfolio regression $b_1$	0.0568	0.0559	0.0539	0.0547	0.0572	0.0556	0.0525 (0.0021)
Portfolio regression $b_2$	-0.0138	-0.0149	-0.0145	-0.0155	-0.0147	-0.0142	-0.0095 (0.0023)
Saving regression $\phi$	0.6480	0.6669	0.6386	0.6542	0.6423	0.6370	0.7300 (0.0500)

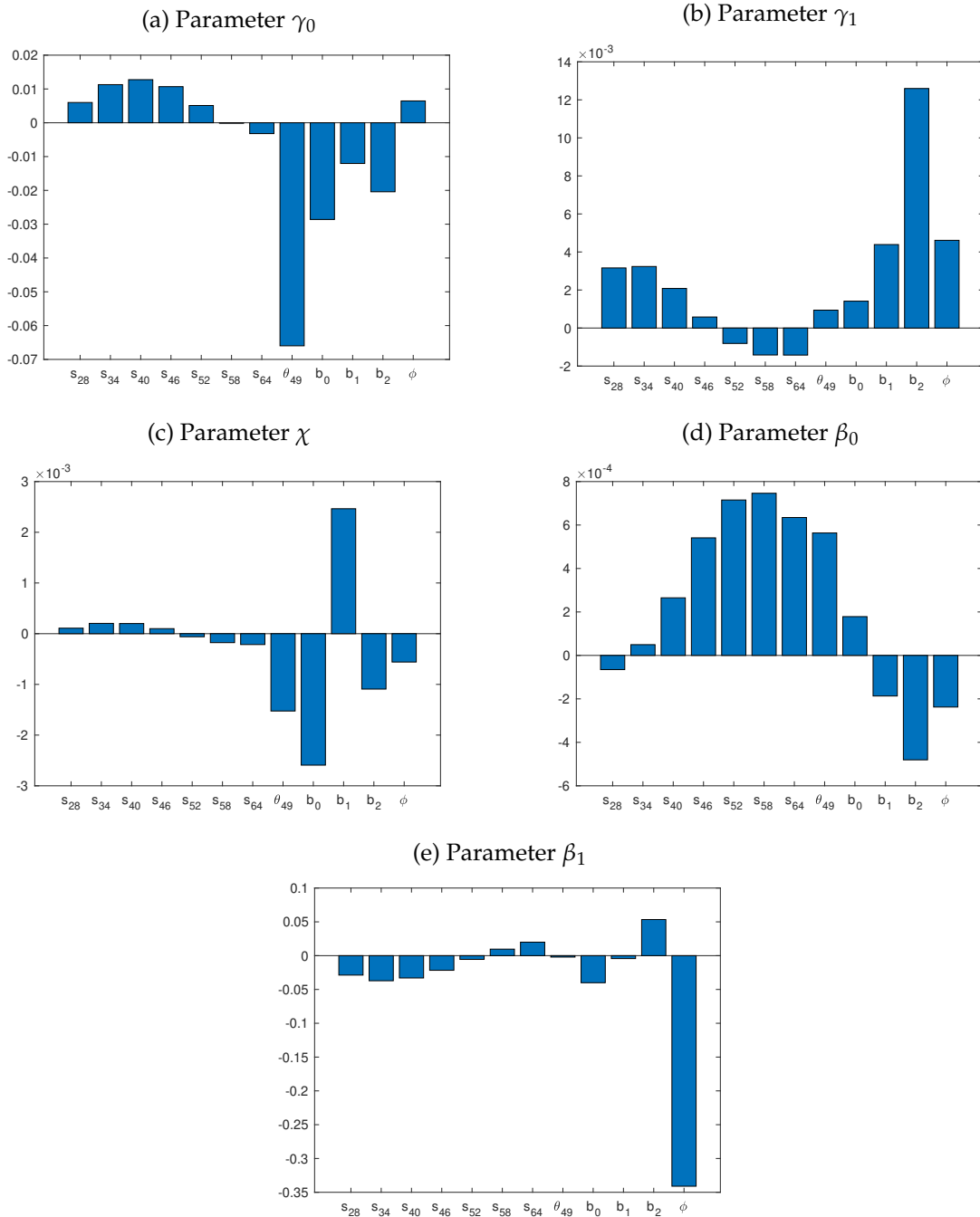
Notes: This table reports the results of structural estimations of alternative versions of the model. I first report the estimated parameter values and their corresponding standard errors, I then indicate which moments were targeted in the estimation, and finally I report the value of the objective and the key moments of the model evaluated at the estimated parameter values.

*Non-homotheticity in risk preferences  $\gamma_1$ .* The degree of non-homotheticity in risk preferences,  $\gamma_1$ , is mainly driven by the regression coefficients  $b_1$  and  $b_2$ . More positive responses to wealth shocks imply a larger elasticity of risk aversion with respect to wealth. The estimate of  $\gamma_1$  is also positively related to the saving regression coefficient  $\phi$ : a high  $\phi$  means relatively more consumption of the permanent-income rich, and one way to get the rich to consume more is by them having bigger average portfolio returns. Since an increase in  $\gamma_1$  means more precautionary savings when young and relatively poor, larger values of  $\gamma_1$  are associated with flatter saving profiles. Therefore, high saving rates for the young increase  $\gamma_1$  and high saving rates for the old decrease  $\gamma_1$ .

*Calvo probability  $\chi$ .* As expected, a high coefficient on passive change  $b_0$  implies a low probability of adjustment  $\chi$ . Since a low probability of adjustment is associated with a flat profile of equity shares over the life cycle, an increase in the equity share at age 50 also lowers the frequency of adjustment. A high regression coefficient  $b_1$  on income growth increases the

estimated probability of adjustment through the extensive margin of portfolio rebalancing.

Figure A.5: Parameter Sensitivity



Notes: This figure shows the sensitivity matrix as defined in Andrews et al. (2017). The values are scaled to correspond to a one standard error increase in each moment.

*Baseline discount rate  $\beta_0$ .* The baseline discount rate is precisely estimated, and is driven mostly by the savings profile of older people (young people are buffer stock savers). Higher saving rates clearly imply that investors are more patient.

*Slope of discount rate  $\beta_1$ .* The slope of the discount rate is only sensitive to changes in the saving regression coefficient  $\phi$ . Obviously, a lower  $\phi$  (more concavity in saving rates over the wealth distribution) is associated with a large heterogeneity in discount rates across initial permanent income levels.

#### A.4.4 Asset Pricing Dynamics

The estimated non-homotheticity in risk tolerance provides qualitative support for asset pricing models based on cross-sectional or time-series variation in risk aversion that quantitatively fit important asset pricing facts such as the equity premium, equity volatility, and countercyclical risk premia (e.g. [Campbell and Cochrane, 1999](#); [Chan and Kogan, 2002](#); [Gârleanu and Panageas, 2015](#)).<sup>35</sup> DRRA preferences have asset pricing implications through two channels. First, households with DRRA have a risk aversion that changes over time as aggregate wealth changes, as in a habit model ([Constantinides, 1990](#); [Campbell and Cochrane, 1999](#)). As a result, cash flow shocks get amplified through their effect on risk aversion. Second, DRRA preferences generate cross-sectional heterogeneity in risk aversion through dispersion in wealth. This heterogeneity leads to differences in optimal portfolios and concentrates holdings in the hands of the most risk tolerant agents. With concentrated equity holdings as in [Mankiw \(1986\)](#), the marginal investor is more exposed to stock market risk than the average household. Ex-ante differences in risk aversion lead to differences in portfolios, which in turn generate variation in the distribution of wealth, thereby changing aggregate risk aversion. In particular, negative (positive) shocks get amplified by a redistribution of wealth to more (less) risk averse agents. Under the right calibration, these effects have been shown to generate empirically relevant magnitudes of the equity premium, equity volatility, and countercyclical risk premia ([Chan and Kogan, 2002](#); [Gârleanu and Panageas, 2015](#)). These two channels amplify the volatility of the stochastic discount factor and lead to a more negative relation between equity investors' marginal utility and equity returns.

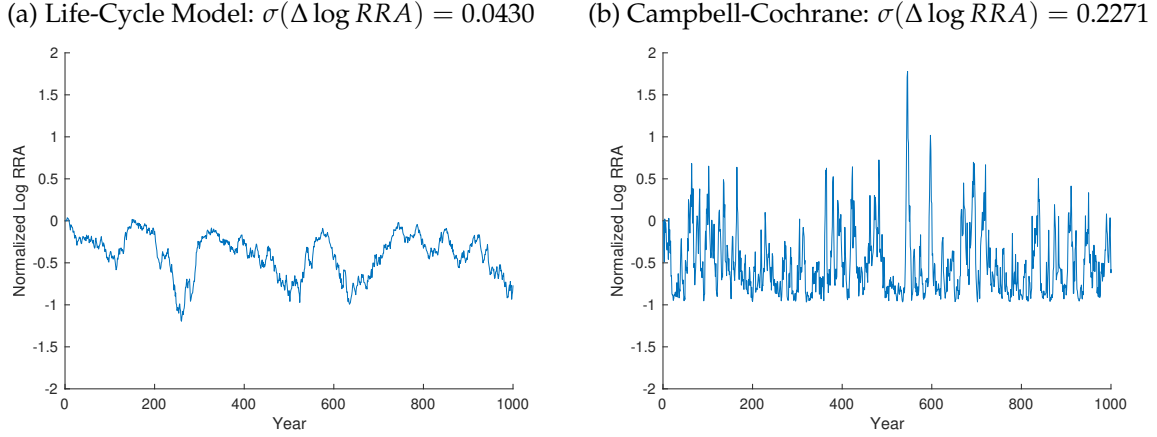
While my estimates do not speak directly to the dynamics of asset prices in general equilibrium, I examine the implications of the estimated DRRA preferences on time-series variation in aggregate risk aversion. Aggregate risk aversion is strongly linked to conditional Sharpe ratios in equilibrium models of asset prices. As a simple illustration, consider an economy where agents choose their portfolio allocations according to the [Merton \(1971\)](#) model. Let  $\kappa$  be the Sharpe ratio and  $\sigma_e$  the volatility of equity. The share of wealth invested in risky assets by individual  $i$  is given by

$$\theta_i = \frac{\kappa}{\gamma_i \sigma_e}. \quad (\text{A.4.7})$$

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<sup>35</sup>Other channels that have been successfully incorporated in general equilibrium asset pricing models include long-run risks ([Bansal and Yaron, 2004](#)), rare disasters ([Rietz, 1988](#); [Barro, 2006](#)), idiosyncratic risk ([Heaton and Lucas, 1996](#); [Constantinides and Duffie, 1996](#)), institutional or intermediary frictions ([Brunnermeier and Sannikov, 2014](#); [He and Krishnamurthy, 2012](#)), and alternative probability assessments due to behavioral mistakes or ambiguity aversion ([Hansen and Sargent, 2001](#)). See [Cochrane \(2017\)](#) for a nice overview.

Figure A.6: Fluctuations in Aggregate Risk Aversion



Notes: This figure illustrates the time-series dynamics of log aggregate risk aversion in simulations of the life-cycle model and the habit model of [Campbell and Cochrane \(1999\)](#).

Imposing market clearing and letting  $N_i$  be the net worth of agent  $i$ , this yields the aggregate relation (see also [Kimball, Shapiro, Shumway, and Zhang, 2019](#))

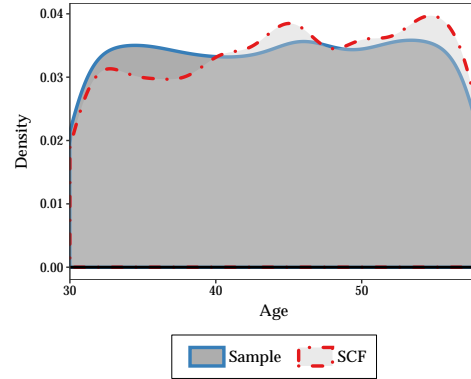
$$\sum_i \theta_i N_i = \frac{\kappa}{\bar{\gamma} \sigma_e} \bar{N} = \bar{N}, \quad \frac{1}{\bar{\gamma}} = \sum_i \frac{N_i}{\bar{N}} \frac{1}{\gamma_i}, \quad (\text{A.4.8})$$

where  $\bar{\gamma}$  is aggregate risk aversion. Assuming a constant stock volatility, it follows that the Sharpe ratio is proportional to aggregate risk aversion.

In model simulations, I find that the volatility of annual changes in log aggregate risk aversion is 4.3%. This is an order of magnitude lower than the variation in aggregate risk aversion implied by [Campbell and Cochrane \(1999\)](#), which is 22.7%. [Figure A.6](#) illustrates these time-series fluctuations in aggregate risk aversion in the models.

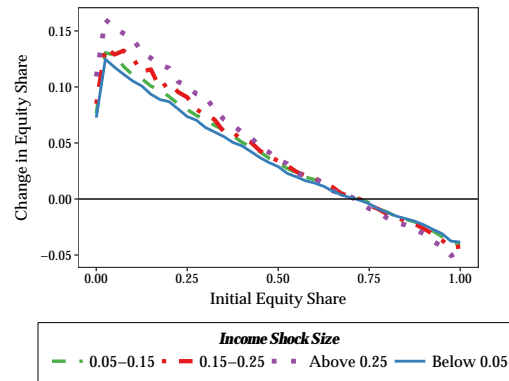
## A.5 Additional Figures and Tables

Figure A.7: Age Distribution



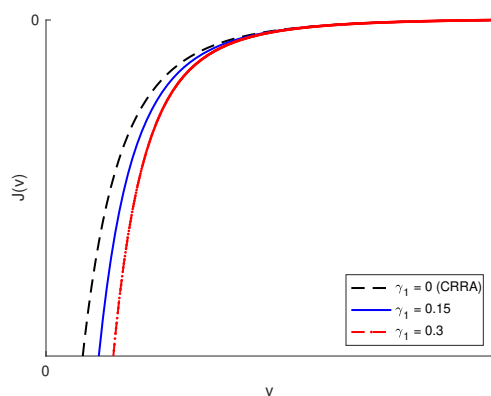
*Notes:* This figure plots the distribution of age in the sample of RI investors versus the distribution of individual retirement wealth for RI investors in the SCF.

Figure A.8: Mean Reversion in Equity Share



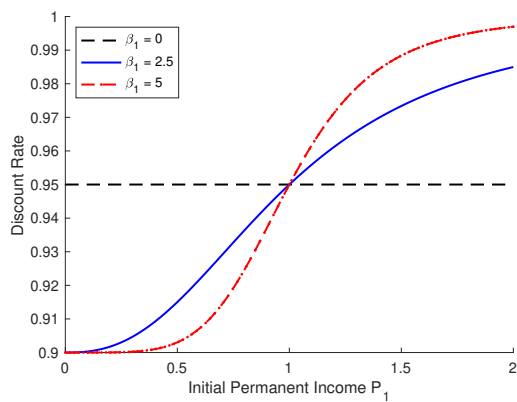
*Notes:* This figure plots the average change in equity share as a function of initial equity share. The sample is split by the magnitude of absolute income growth.

Figure A.9: Modeled Certainty Equivalent Aggregator



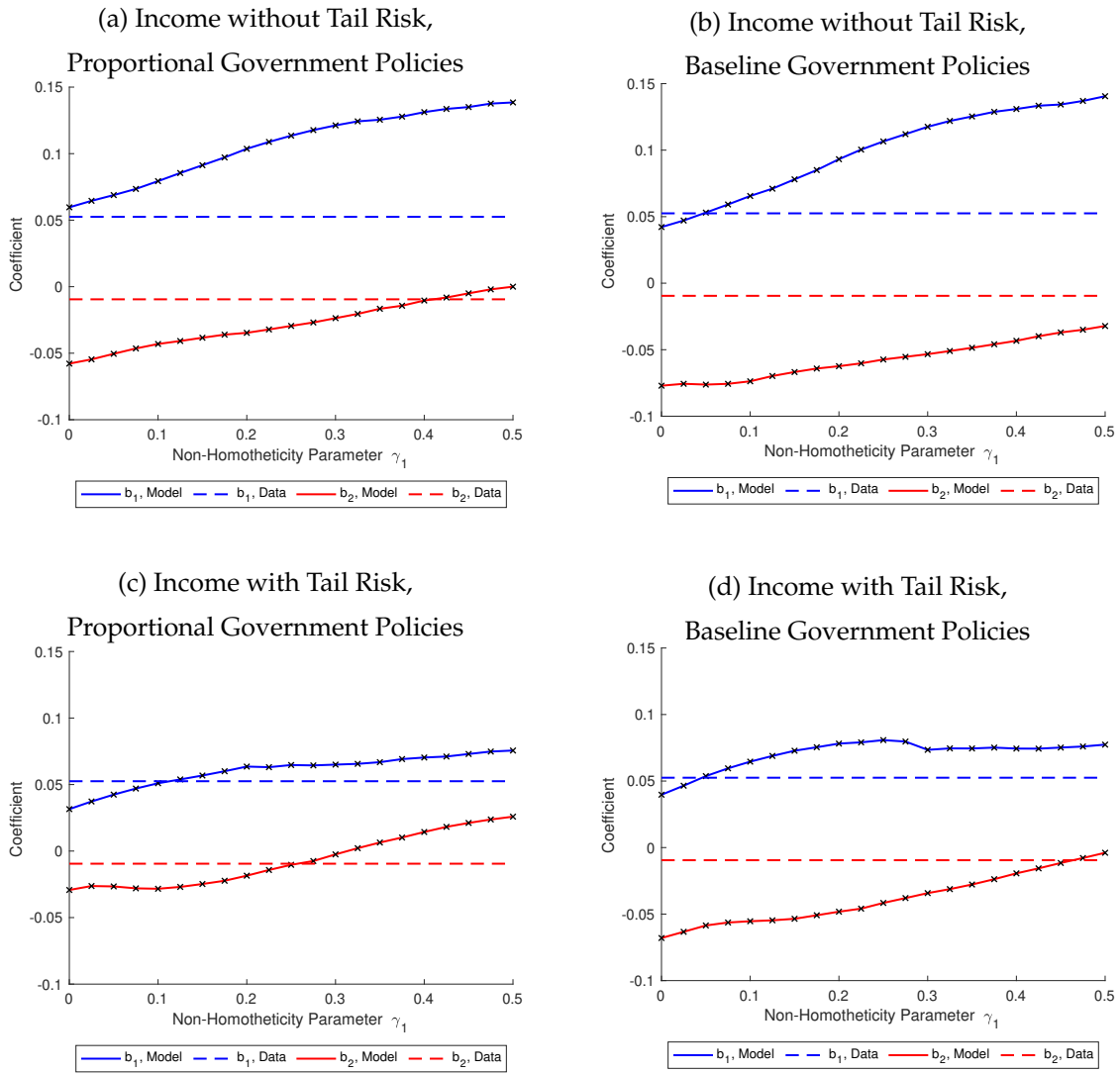
Notes: This figure plots the certainty equivalent aggregator  $J$  that is the solution to the ODE (24) with baseline risk aversion  $\gamma_0 = 5$  and for different values of the non-homotheticity parameter  $\gamma_1$ .

Figure A.10: Modeled Discount Factor Heterogeneity by Initial Permanent Income



Notes: This figure plots the parameterization (25) for discount factors  $\beta$  that are a function of initial permanent income  $P_1$ , for different values of the slope parameter  $\beta_1$ .

Figure A.11: Adjustment Model Regression Coefficients in Life-Cycle Model



Notes: This figure plots the coefficients  $b_1$  and  $b_2$  of the portfolio adjustment regression estimated in model-simulated data for different values of  $\gamma_1$ , compared to the empirical estimates of  $b_1$  and  $b_2$ , across different model specifications.



Table A.10: Income and Equity Shares – Long-Run Effects

	$\Delta_{3+j} \log \text{equity share}_{t+j}$				Log equity share <sub>t</sub>	
	$j = 0$	$j = 1$	$j = 3$	$j = 5$	(5)	(6)
	(1)	(2)	(3)	(4)		
$\Delta_3 \log \text{income}_t$	0.0208 (0.0011)	0.0250 (0.0012)	0.0224 (0.0017)	0.0203 (0.0025)		
Log income <sub>t</sub>					0.0313 (0.0004)	0.0388 (0.0005)
Employer × year FE	Y	Y	Y	Y	Y	Y
Log equity share <sub>t-3</sub>	Y	Y	Y	Y		
Demographic controls	Y	Y	Y	Y	Y	Y
Sample						Non-default investor
R-squared	0.272	0.297	0.370	0.377	0.090	0.076
Share of individuals	53.4%	51.6%	37.9%	27.0%	97.3%	51.5%

*Notes:* Columns (1)–(4) present estimates of the long-run relation between income and equity shares by reporting the results of an OLS regression of changes in log equity shares, measured over several horizons, on three-year income growth. The demographic controls include a second-order polynomial in age, gender, marital status, a second-order polynomial in employment tenure, log income, and the log of financial assets, all measured at  $t - 3$ . Columns (5)–(6) report the results of an OLS cross-sectional regression of log equity shares on log income. The demographic controls include a second-order polynomial in age, gender, marital status, and a second-order polynomial in employment tenure. Standard errors are clustered at the individual level.